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A STUDY ON HESSENBERG TYPE-2 TRIANGULAR FUZZY MATRICES

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Abstract:

The concept of a type-2 fuzzy set was introduced by Zadeh as an extension of an ordinary fuzzy set. Type-2 fuzzy sets have grades of membership that are themselves fuzzy. Hence the membership function of a type-2 fuzzy set is three dimensional, and it is the new third dimension that provides new design degrees of freedom for handling uncertainties. Also fuzzy matrices play an important role in scientific developments. In this paper, the concept of Hessenberg type-2 triangular fuzzy matrices is proposed. Also some properties of Hessenberg type-2 triangular fuzzy matrices are presented.

Key words: Type-2 fuzzy set, Type-2 triangular fuzzy number, Type-2 triangular fuzzy matrices.

1. INTRODUCTION

Fuzzy methods allow the processing of imprecise and indecisive variables. Fuzzy methods come in two main flavors, type-1 and type-2. The concept of a type-2 fuzzy set, which is an extension of the concept of an ordinary fuzzy set, was introduced by Zadeh [13]. Type-1 fuzziness represents the first step toward the full processing of qualms and the type-2 fuzziness builds upon the strengths of the type-1 methods and overcomes some of the limitations. It is observed that the type-2 fuzzy paradigm is a lesser known and a lesser exploited area of fuzziness and also the type-2 fuzzy methods provide second order uncertainties allowing fuzzy systems to accurately deal with real world uncertainty. At present the full potential of type-2 methods has not been exploited by practitioners, probably because of computational expense of type-2 operations. In the current climate of ever faster, more dominant and more affordable hardware, the type-2 fuzzy methods present a

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stimulating opportunity to explore uncertainties in real world systems in ways that were not previously possible.

Fuzzy matrices were introduced for the first time by Thomason [11], who discussed the convergence of powers of fuzzy matrix. Kim [3] presented some important results on determinant of square fuzzy matrices. Ragab et al [6] presented some properties of the min-max composition of fuzzy matrices. Fuzzy matrices play an important role in scientific developments. Shyamal and Pal [7] presented triangular fuzzy matrices. Stephen Dinagar and Anbalagan [8] presented new ranking function and arithmetic operations on generalized type-2 trapezoidal fuzzy numbers. Stephen Dinagar and Latha [9] introduced type-2 triangular fuzzy matrices. In[10] they also presented some types and properties of type-2 triangular fuzzy matrices.

This paper is organized as follows: Firstly in section-2 of this paper, we recall the definition of type-2 triangular fuzzy number and arithmetic operations on type-2 triangular fuzzy numbers. In section-3, we review the definition of type-2 triangular fuzzy matrices (T2TFM) and some operations on T2TFMs. In section-4, we review types of type-2 triangular fuzzy matrices. In section-5, we define Hessenberg T2TFMs. In section-6, we derive some properties of Hessenberg T2TFMs. Finally in section-7, conclusion is also included.

2. TYPE-2 TRIANGULAR FUZZY NUMBERS

2.1. Definition: Fuzzy set

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set A in a universe of discourse X is defined as the following set of pairs:

$$A = \{(x, \mu_A(x)); x \in X\}.$$

Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2. Definition: (Zadeh) Type-2 fuzzy set

A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0,1].

2.3. Definition:

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The type-2 fuzzy sets are defined by functions of the form μ_A: x → χ ([0,1]) where χ ([0,1]) denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1]. An example [4] of a membership function of this type is given in the following figure.

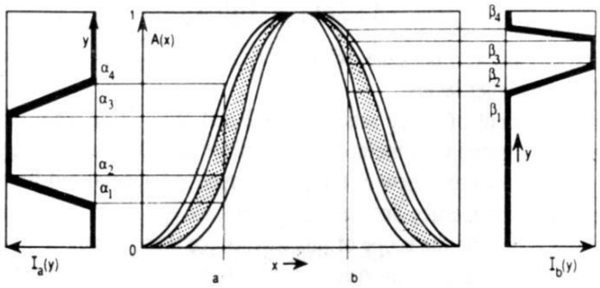

Fig. Illustration of the concept of a fuzzy set of type-2.

2.4. Definition: Type-2 fuzzy number [7]

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

(i) $\tilde{\tilde{A}}$ is normal,

(ii) $\tilde{\tilde{A}}$ is a convex set,

(iii) The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

2.5. Definition: Type-2 triangular fuzzy number

A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ on R is given by $\tilde{\tilde{A}}$ = $\{(x,(\mu_A^1(x),\mu_A^2(x),\mu_A^3(x)); x\epsilon R\}$ and $\mu_A^1(x) \leq \mu_A^2(x) \leq \mu_A^3(x)$, for all x϶ R. Denote $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, where $\tilde{A}_1$ = (A_1^L,A_1^N,A_1^U), $\tilde{A}_2 = (A_2^L,A_2^N,A_2^U)$ and $\tilde{A}_3 = (A_3^L,A_3^N,A_3^U)$ are same type of fuzzy numbers.

2.6. Arithmetic operations on type-2 triangular fuzzy numbers [9]

Let $\tilde{\tilde{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) = ((a_1^L,a_1^N,a_1^U),(a_2^L,a_2^N,a_2^U),(a_3^L,a_3^N,a_3^U))$ and $\tilde{\tilde{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = ((b_1^L,b_1^N,b_1^U),(b_2^L,b_2^N,b_2^U),(b_3^L,b_3^N,b_3^U))$ be two type-2 triangular fuzzy numbers. Then we define,

(i)Addition:

$\tilde{\tilde{a}} + \tilde{\tilde{b}} = ((a_1^L+b_1^L,a_1^N+b_1^N,a_1^U+b_1^U),(a_2^L+b_2^L,a_2^N+b_2^N,a_2^U+b_2^U),(a_3^L+b_3^L,a_3^N+b_3^N, a_3^U+b_3^U))$

(ii)Subtraction:

$\tilde{\tilde{a}} - \tilde{\tilde{b}} = ((a_1^L-b_3^U,a_1^N-b_3^N,a_1^U-b_3^L),(a_2^L-b_2^U,a_2^N-b_2^N,a_2^U-b_2^L),(a_3^L-b_1^U,a_3^N-b_1^N, a_3^U-b_1^L))$

(iii)Scalar multiplication:

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If $k \geq 0$ and $k \in R$ then $k\tilde{\tilde{a}}$ = $((ka_1^L,ka_1^N,ka_1^U),(ka_2^L,ka_2^N,ka_2^U),(ka_3^L,ka_3^N,ka_3^U))$ and if $k < 0$ and $k \in R$ then $k\tilde{\tilde{a}}$ = $((ka_3^U,ka_3^N,ka_3^L),(ka_2^U,ka_2^N,ka_2^L),(ka_1^U,ka_1^N,ka_1^L))$.

(iv)Multiplication:

Define $\sigma b = b_1^L+b_1^N+b_1^U+b_2^L+b_2^N+b_2^U+b_3^L+b_3^N+b_3^U$.If $\sigma b \geq 0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_1^L\sigma b}{9}, \frac{a_1^N\sigma b}{9}, \frac{a_1^U\sigma b}{9}\right), \left(\frac{a_2^L\sigma b}{9}, \frac{a_2^N\sigma b}{9}, \frac{a_2^U\sigma b}{9}\right), \left(\frac{a_3^L\sigma b}{9}, \frac{a_3^N\sigma b}{9}, \frac{a_3^U\sigma b}{9}\right) \right).$$

If $\sigma b<0$, then

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\frac{a_3^U\sigma b}{9}, \frac{a_3^N\sigma b}{9}, \frac{a_3^L\sigma b}{9}\right), \left(\frac{a_2^U\sigma b}{9}, \frac{a_2^N\sigma b}{9}, \frac{a_2^L\sigma b}{9}\right), \left(\frac{a_1^U\sigma b}{9}, \frac{a_1^N\sigma b}{9}, \frac{a_1^L\sigma b}{9}\right) \right).$$

(v)Division:

Whenever $\sigma b \neq 0$ we define division as follows:If $\sigma b>0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{9a_1^L}{\sigma b}, \frac{9a_1^N}{\sigma b}, \frac{9a_1^U}{\sigma b}\right), \left(\frac{9a_2^L}{\sigma b}, \frac{9a_2^N}{\sigma b}, \frac{9a_2^U}{\sigma b}\right), \left(\frac{9a_3^L}{\sigma b}, \frac{9a_3^N}{\sigma b}, \frac{9a_3^U}{\sigma b}\right) \right).$$

If $\sigma b<0$, then

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left(\left(\frac{9a_3^U}{\sigma b}, \frac{9a_3^N}{\sigma b}, \frac{9a_3^L}{\sigma b}\right), \left(\frac{9a_2^U}{\sigma b}, \frac{9a_2^N}{\sigma b}, \frac{9a_2^L}{\sigma b}\right), \left(\frac{9a_1^U}{\sigma b}, \frac{9a_1^N}{\sigma b}, \frac{9a_1^L}{\sigma b}\right) \right).$$

2.7. The proposed ranking function [9]

Let F(R) be the set of all type-2 normal triangular fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function Ř:F(R) → R which maps each fuzzy number into R.

Suppose if $\tilde{\tilde{A}} = (\tilde{A}_1,\tilde{A}_2,\tilde{A}_3)$, = $((A_1^L,A_1^N,A_1^U),(A_2^L,A_2^N,A_2^U),(A_3^L,A_3^N,A_3^U))$ then we define $\check{R}(\tilde{\tilde{A}}) = (A_1^L+A_1^N+A_1^U+A_2^L+A_2^N+A_2^U+A_3^L+A_3^N+A_3^U) / 9$.

Also we define orders on F(R) by

$\check{R}(\tilde{\tilde{A}}) \geq \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \gtrsim_{\check{R}} \tilde{\tilde{B}}$,

$\check{R}(\tilde{\tilde{A}}) \leq \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \lesssim_{\check{R}} \tilde{\tilde{B}}$

and $\check{R}(\tilde{\tilde{A}}) = \check{R}(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} =_{\check{R}} \tilde{\tilde{B}}$.

2.8. Definition: Equal & Equivalent Fuzzy Numbers

Let $\tilde{\tilde{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$ = $((a_1^L,a_1^N,a_1^U),(a_2^L,a_2^N,a_2^U),(a_3^L,a_3^N,a_3^U))$ and $\tilde{\tilde{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ = $((b_1^L,b_1^N,b_1^U),(b_2^L,b_2^N,b_2^U),(b_3^L,b_3^N,b_3^U))$ be two type-2 triangular fuzzy numbers. Then $\tilde{\tilde{a}}$ and $\tilde{\tilde{b}}$ are said to be equal if $a_1^L = b_1^L$, $a_1^N = b_1^N$, $a_1^U = b_1^U$, $a_2^L = b_2^L$, $a_2^N = b_2^N$, $a_2^U = b_2^U$,

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$a_3^L = b_3^L$, $a_3^N = b_3^N$, $a_3^U = b_3^U$. It is denoted by $\tilde{\tilde{a}} = \tilde{\tilde{b}}$. Suppose if $\check{R}(\tilde{\tilde{a}}) = \check{R}(\tilde{\tilde{b}})$ then $\tilde{\tilde{a}}$ and $\tilde{\tilde{b}}$ are said to be equivalent fuzzy numbers. It is denoted by $\tilde{\tilde{a}} \approx \tilde{\tilde{b}}$.

2.9. Definition: Type-2 zero triangular fuzzy number

If $\tilde{\tilde{A}}$ = ((0,0,0),(0,0,0),(0,0,0)) then $\tilde{\tilde{A}}$ is said to be a type-2 zero triangular fuzzy number. It is denoted by 0.

2.10. Definition: Type-2 zero-equivalent triangular fuzzy number

A type-2 triangular fuzzy number $\tilde{\tilde{A}}$ is said to be a type-2 zero-equivalent triangular fuzzy number if $\check{R}$ ($\tilde{\tilde{A}}$) = 0. It is denoted by $\tilde{\tilde{0}}$.

3. TYPE-2 TRIANGULAR FUZZY MATRICES (T2TFMs) [9]

3.1. Definition: Type-2 triangular fuzzy matrix (T2TFM)

A type-2 triangular fuzzy matrix (T2TFM) of order m×n is defined as A = $(\tilde{\tilde{a}}_{ij})_{mxn}$ where the ij^{th} element $\tilde{\tilde{a}}_{ij}$ of A is the type-2 triangular fuzzy number.

3.2. Operations onT2TFMs:

As for classical matrices we define the following operations on T2TFMs. Let A = $(\tilde{\tilde{a}}_{ij})$ and B = $(\tilde{\tilde{b}}_{ij})$ be two T2TFMs of same order. Then we have the following:

(i) A+B = $(\tilde{\tilde{a}}_{ij} + \tilde{\tilde{b}}_{ij})$

(ii) A−B = $(\tilde{\tilde{a}}_{ij} - \tilde{\tilde{b}}_{ij})$

(iii) For A = $(\tilde{\tilde{a}}_{ij})_{mxn}$ and B = $(\tilde{\tilde{b}}_{ij})_{nxk}$ then AB = $(\tilde{\tilde{c}}_{ij})_{mxk}$ where $\tilde{\tilde{c}}_{ij} = \sum_{p=1}^{n} \tilde{\tilde{a}}_{ip} . \tilde{\tilde{b}}_{pj}$, i=1,2,…..,m and j=1,2,…..,k.

(iv) A^T or A′= $(\tilde{\tilde{a}}_{ji})$

(v) kA = $(k\tilde{\tilde{a}}_{ij})$, where k is a scalar.

4. TYPES OF TYPE-2 TRIANGULAR FUZZY MATRICES

4.1. Definition: Upper triangular T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called an upper triangular T2TFM if all the entries below the principal diagonal are 0.

4.2. Definition: Upper triangular - equivalent T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called an upper triangular - equivalent T2TFM if all the entries below the principal diagonal are $\tilde{\tilde{0}}$.

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4.3. Definition: Lower triangular T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called a lower triangular T2TFM if all the entries above the principal diagonal are 0.

4.4. Definition: Lower triangular - equivalent T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called a lower triangular - equivalent T2TFM if all the entries above the principal diagonal are $\tilde{\tilde{0}}$.

4.5. Definition: Triangular T2TFM

A square T2TFM A = $(\tilde{\tilde{a}}_{ij})$ is called a triangular T2TFM if it is either upper triangular T2TFM or lower triangular T2TFM.

4.6. Definition: Triangular - equivalent T2TFM

A square T2TFM A = $(\tilde{\tilde{a}}_{ij})$ is called a triangular - equivalent T2TFM if it is either upper triangular – equivalent T2TFM or lower triangular - equivalent T2TFM.

5. HESSENBERG TYPE-2 TRIANGULAR FUZZY MATRICES

5.1. Definition: Upper Hessenberg T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called an upper Hessenberg T2TFM if all the entries below the first sub diagonal are 0.

5.2. Definition: Upper Hessenberg - equivalent T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called an upper Hessenberg - equivalent T2TFM if all the entries below the first sub diagonal are $\tilde{\tilde{0}}$.

5.3. Definition: Lower Hessenberg T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called a lower Hessenberg T2TFM if all the entries above the first super diagonal are 0.

5.4. Definition: Lower Hessenberg - equivalent T2TFM

A square T2TFM A= $(\tilde{\tilde{a}}_{ij})$ is called a lower Hessenberg - equivalent T2TFM if all the entries above the first super diagonal are $\tilde{\tilde{0}}$.

5.5. Definition: Hessenberg T2TFM

A square T2TFM A = $(\tilde{\tilde{a}}_{ij})$ is called a Hessenberg T2TFM if it is either upper Hessenberg T2TFM or lower Hessenberg T2TFM.

VOL: 2, ISSUE 1, JANUARY-MARCH 2026
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6. PROPERTIES OF HESSENBERG TYPE-2 TRIANGULAR FUZZY MATRICES

Property 6.1

The sum of two upper Hessenberg T2TFMs of order n is also an upper Hessenberg T2TFM of order n.

Proof:

Let A = ($\tilde{\tilde{a}}_{ij}$) and B = ($\tilde{\tilde{b}}_{ij}$) be two upper Hessenberg T2TFMs of order n. Since A and B are upper Hessenberg T2TFMs, $\tilde{\tilde{a}}_{ij}$ = 0 and $\tilde{\tilde{b}}_{ij}$ = 0 for all i $>$ j+1 ; i, j = 1, 2, …, n.

Let A+B = C. Then ($\tilde{\tilde{a}}_{ij} + \tilde{\tilde{b}}_{ij}$) = ($\tilde{\tilde{c}}_{ij}$).

For all i $>$ j+1; i, j = 1, 2, …, n, $\tilde{\tilde{c}}_{ij}$ = $\tilde{\tilde{a}}_{ij} + \tilde{\tilde{b}}_{ij}$ = 0 + 0 = 0.

Hence C is an upper Hessenberg T2TFM of order n.

Property 6.2

The sum of two lower Hessenberg T2TFMs of order n is also a lower Hessenberg T2TFM of order n.

Proof:

Let A = ($\tilde{\tilde{a}}_{ij}$) and B = ($\tilde{\tilde{b}}_{ij}$) be two lower Hessenberg T2TFMs of order n. Since A and B are lower Hessenberg T2TFMs, $\tilde{\tilde{a}}_{ij}$ = 0 and $\tilde{\tilde{b}}_{ij}$ = 0 for all i+1 $<$ j ; i, j = 1, 2, …, n.

Let A+B = C. Then ($\tilde{\tilde{a}}_{ij} + \tilde{\tilde{b}}_{ij}$) = ($\tilde{\tilde{c}}_{ij}$).

For all i+1$<$ j ; i, j = 1, 2, …, n, $\tilde{\tilde{c}}_{ij}$ = $\tilde{\tilde{a}}_{ij} + \tilde{\tilde{b}}_{ij}$ = 0 + 0 = 0.

Hence C is a lower Hessenberg T2TFM of order n.

Property 6.3

The scalar multiplication of an upper Hessenberg T2TFM is also an upper Hessenberg T2TFM.

Proof:

Let A = ($\tilde{\tilde{a}}_{ij}$) be an upper Hessenberg T2TFM. Since A is an upper Hessenberg T2TFM, $\tilde{\tilde{a}}_{ij}$ = 0 for all i $>$ j+1 ; i, j = 1, 2, …, n.

Let k be a scalar and kA = B. Then (k$\tilde{\tilde{a}}_{ij}$) = ($\tilde{\tilde{b}}_{ij}$).

For all i $>$ j+1 ; i, j = 1, 2, …, n, $\tilde{\tilde{b}}_{ij}$= k$\tilde{\tilde{a}}_{ij}$ = k0 = 0.

Hence B is also an upper Hessenberg T2TFM.

Property 6.4

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The scalar multiplication of a lower Hessenberg T2TFM is also a lower Hessenberg T2TFM.

Proof:

Let A = $(\tilde{\tilde{a}}_{ij})$ be a lower Hessenberg T2TFM. Since A is a lower Hessenberg T2TFM, $\tilde{\tilde{a}}_{ij}$ = 0 for all i+1 $<$ j ; i, j = 1, 2, …, n.

Let k be a scalar and kA = B. Then $(k\tilde{\tilde{a}}_{ij}) = (\tilde{\tilde{b}}_{ij})$.

For all i+1 $<$ j ; i, j = 1, 2, …, n, $\tilde{\tilde{b}}_{ij} = k\tilde{\tilde{a}}_{ij}$ = k0 = 0.

Hence B is also a lower Hessenberg T2TFM.

Property 6.5

If A=$(\tilde{\tilde{a}}_{ij})$ is the upper Hessenberg T2TFM of order n and B=$(\tilde{\tilde{b}}_{ij})$ is the upper triangular T2TFM of order n then AB is the upper Hessenberg T2TFM of order n .

Proof:

Given that A = $(\tilde{\tilde{a}}_{ij})$ is the upper Hessenberg T2TFM of order n and B=$(\tilde{\tilde{b}}_{ij})$ is the upper triangular T2TFM of order n.

Since A is the upper Hessenberg T2TFM, $\tilde{\tilde{a}}_{ij}$ = 0 for all i $>$ j+1 ; i, j = 1, 2, …, n and also B is the upper triangular T2TFM $\tilde{\tilde{b}}_{ij}$ = 0 for all i $>$ j ; i, j = 1, 2, …, n.

Let AB = C = $(\tilde{\tilde{c}}_{ij})$ where $\tilde{\tilde{c}}_{ij} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \,.\, \tilde{\tilde{b}}_{kj}$.

Now we will show that $\tilde{\tilde{c}}_{ij}$ = 0 for all i $>$ j+1 ; i, j = 1, 2, …, n.

For i $>$ j+1 we have $\tilde{\tilde{a}}_{ik}$ = 0 for k = 1, 2, …, (i−2), and similarly $\tilde{\tilde{b}}_{kj}$ = 0 for k = (i−1), i, (i+1), …, n.

Therefore $\tilde{\tilde{c}}_{ij} = \sum_{k=1}^{n} \tilde{\tilde{a}}_{ik} \,.\, \tilde{\tilde{b}}_{kj}$

$$= \sum_{k=1}^{(i-2)} \tilde{\tilde{a}}_{ik} \,.\, \tilde{\tilde{b}}_{kj} + \sum_{k=(i-1)}^{n} \tilde{\tilde{a}}_{ik} \,.\, \tilde{\tilde{b}}_{kj}$$

$$= \sum_{k=1}^{(i-2)} 0 \,.\, \tilde{\tilde{b}}_{kj} + \sum_{k=(i-1)}^{n} \tilde{\tilde{a}}_{ik} \,.\, 0$$

$$= 0$$

Hence the result follows.

Property 6.6

The transpose of an upper Hessenberg T2TFM of order n is a lower Hessenberg T2TFM of order n and vice versa.

Proof:

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Let A = ($\tilde{\tilde{a}}_{ij}$) be an upper Hessenberg T2TFM of order n. Since A is an upper Hessenberg T2TFM, $\tilde{\tilde{a}}_{ij}$ = 0 for all i > j+1 ; i, j = 1, 2, …, n.

Let B be the transpose of A. Then A′ = B. i.e, ($\tilde{\tilde{a}}_{ji}$) = ($\tilde{\tilde{b}}_{ij}$).

For all i > j+1; i, j = 1, 2, …, n, $\tilde{\tilde{a}}_{ij}$ = 0 = $\tilde{\tilde{b}}_{ji}$.

That is for all i+1 < j ; i, j = 1, 2, …, n, $\tilde{\tilde{b}}_{ij}$ = 0.

Hence B is a lower Hessenberg T2TFM of order n.

Remark:

A matrix that is both upper Hessenberg T2TFM and lower Hessenberg T2TFM is a tridiagonal T2TFM.

7. CONCLUSION

In this article Hessenberg type-2 triangular fuzzy matrices are defined and also some properties of Hessenberg T2TFMs are proved. Many fuzzy linear algebra algorithms require significantly less computational effort when applied to triangular T2TFM, and this improvement often carries over to Hessenberg T2TFM as well. So the discussed notions are very useful to solve the system of fuzzy linear equations. The theories of the discussed T2TFMs may be utilized in further works.

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