RCHUB: JOURNAL OF COMPUTATIONAL INTELLIGENCE SCIENCE AND ENGINEERING (JCISE)
VOL: 2, ISSUE 1, JANUARY-MARCH 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

Comparative Study of Max–Min–Max, Improved, and Refined Composite Relations in Fermatean *m*-Polar Fuzzy Sets

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Abstract

In this paper, we explore the fermatean m-polar fuzzy max-min-max composite relation, improved fermatean m-polar fuzzy composite relation and propose refined fermatean m-polar fuzzy composite relation approach for better results. The validity of the refined fermatean m-polar fuzzy composite relation is tested through numerical experiments, comparing it to the traditional fermatean m-polar fuzzy max-min-max composite relation and improved fermatean m-polar fuzzy composite relation. The results demonstrate that the refined fermatean m-polar fuzzy composite relation provides superior output. In conclusion, the refined Fermatean m-polar fuzzy composite relation provides a powerful and flexible approach for handling uncertainty in real-life decision-making problems. By incorporating higher degrees of membership, non-membership, and hesitation, this method enhances accuracy and reliability in complex scenarios such as medical diagnosis, risk assessment, and multi-criteria decision-making. Its ability to process vague and imprecise information makes it a valuable tool for decision-makers, ensuring more informed and rational choices across various fields.

Keywords: Fuzzy set, Fermatean *m*-Polar Fuzzy Set, Fermatean *m*-Polar Fuzzy max-min- max composite relation, improved fermatean *m*-polar fuzzy composite relation, refined fermatean *m*-polar fuzzy composite relation and applications.

Introduction:

Fuzzy set theory was introduced by Zadeh **[15]**. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory, etc. This set helps us to solve many real-life problems. De Beats **[4]** introduced the relations in fuzzy sets. Atanassov **[1]** included the non-membership function in the fuzzy set and make a new fuzzy set is

VOL: 2, ISSUE 1, JANUARY-MARCH 2026
RCHUB PUBLISHER
ISSN: 3107-8168
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intuitionistic fuzzy set. Biswas **[2]** introduced the formal framework of intuitionistic fuzzy relations, extending classical fuzzy relations by incorporating membership, non-membership, and hesitation degrees to better model uncertainty in relational structures. Yager **[14]** introduced the Pythagorean fuzzy sets (PF-sets) as an extension of is intuitionistic fuzzy set. Ejegwa **[5][6][7]** make the max min composition rule in the intuitionistic fuzzy relations and apply in this rule in appointment of positions in an organization and extend the composition relation to improved intuitionistic fuzzy composition for getting the better result in decision making problems and also introduced modified composition relation in Fermatean fuzzy set and use this concept solved pattern recognition problem. Senapati. T. Yager **[13]** presented the notion of Fermatean fuzzy sets. Sabu **[12]** was first defining the Multi Fuzzy Sets. Chen **[9]** was introduced the m-Polar Fuzzy Sets. Ghous Ali **[8]** presented some decision-making under Fermatean fuzzy bipolar soft framework. Naeem **[10]** was introduced Pythagorean m-polar fuzzy set and investigated some novel features of Pythagorean m-polar fuzzy sets with applications. The notion of Pythagorean m-polar Fuzzy Neutrosophic Topology with Applications were presented by Atiqa Siraj **[3].** Radharamani and Rajeswari **[11]** introduced the fermatean *m*-polar fuzzy set is the extension of the Pythagorean m-polar fuzzy sets.

In this paper we use the max-min-max composite relation, improved composite relation and refined composite relation in the fermatean *m*-polar fuzzy set and compare these three relational values and apply in the decision-making problems.

2.Preliminaries

Definition: 2.1 [15]

Let U be universal set. A *fuzzy set* F on U is defined by $F = \{\langle u, \varphi(u)\rangle; for\ all\ u \in U\}$, where φ is a function from U to [0,1] and it is called the membership function.

Definition :2.2 [1]

Let U be universal set. An *intuitionistic fuzzy set* F on U is defined by

$F = \{\langle u, (\varphi(u), \Psi(u))\rangle; \forall\ u \in U\}$, where the membership function $\varphi: U \to [0,1]$ and the non membership function $\Psi: U \to [0,1]$ which satisfies the condition $\varphi(u) + \Psi(u) \leq 1, \forall u \in U$.

Definition :2.3 [14]

Let U be universal set. A *Pythagorean fuzzy set* F on U is defined by

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$F = \{\langle u, (\varphi(u), \Psi(u))\rangle; \forall\, u \in U\}$, where the membership function $\varphi: U \to [0,1]$ and the non membership function $\Psi: U \to [0,1]$ which satisfies the condition

$(\varphi(u))^2 + (\Psi(u))^2 \leq 1, \forall u \in U.$

Definition :2.4 [13]

Let U be universal set. An *Fermatean fuzzy set* F on U is defined by

$F = \{\langle u, (\varphi(u), \Psi(u))\rangle; \forall\, u \in U\}$, where the membership function $\varphi: U \to [0,1]$ and the non membership function $\Psi: U \to [0,1]$ which satisfies the condition

$(\varphi(u))^3 + (\Psi(u))^3 \leq 1, \forall u \in U.$

Definition :2.5 [10]

Let 'm' be any positive integer. A Pythagorean m-polar fuzzy set (PmPFS) on U is defined by

$P = \left\{\langle u, \left(\varphi_P^{(1)}(u), \Psi_P^{(1)}(u)\right), \left(\varphi_P^{(2)}(u), \Psi_P^{(2)}(u)\right), \dots, \left(\varphi_P^{(m)}(u), \Psi_P^{(m)}(u)\right)\rangle; \forall\, u \in U\right\}$,

where $\varphi_P^{(i)}: U \to [0,1]$ is the membership function $\Psi_P^{(i)}: U \to [0,1]$ is the non membership function which satisfies the condition

$$\left(\varphi_P^{(i)}(u)\right)^2 + \left(\Psi_P^{(i)}(u)\right)^2 \leq 1, \forall u \in U, i = 1,2,3,\dots,m.$$

The set

$P = \left\{\langle u, \left(\varphi_P^{(1)}(u), \Psi_P^{(1)}(u)\right), \left(\varphi_P^{(2)}(u), \Psi_P^{(2)}(u)\right), \dots, \left(\varphi_P^{(m)}(u), \Psi_P^{(m)}(u)\right)\rangle; \forall\, u \in U\right\}$ is rewrite as $P = \left\{\langle u, \left(\varphi_P^{(i)}(u), \Psi_P^{(i)}(u)\right)\rangle; i = 1,2,3,\dots,m \text{ and } \forall\, u \in U\right\}$.

Table 1

U				
u_1	$\left(\varphi_P^{(1)}(u_1), \Psi_P^{(1)}(u_1)\right)$	$\left(\varphi_P^{(2)}(u_1), \Psi_P^{(2)}(u_1)\right)$	…………….	$\left(\varphi_P^{(m)}(u_1), \Psi_P^{(m)}(u_1)\right)$
u_2	$\left(\varphi_P^{(1)}(u_2), \Psi_P^{(1)}(u_2)\right)$	$\left(\varphi_P^{(2)}(u_2), \Psi_P^{(2)}(u_2)\right)$	…………….	$\left(\varphi_P^{(m)}(u_2), \Psi_P^{(m)}(u_2)\right)$
u_3	$\left(\varphi_P^{(1)}(u_3), \Psi_P^{(1)}(u_3)\right)$	$\left(\varphi_P^{(2)}(u_3), \Psi_P^{(2)}(u_3)\right)$	…………….	$\left(\varphi_P^{(m)}(u_3), \Psi_P^{(m)}(u_3)\right)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_k	$\left(\varphi_P^{(1)}(u_k), \Psi_P^{(1)}(u_k)\right)$	$\left(\varphi_P^{(2)}(u_k), \Psi_P^{(2)}(u_k)\right)$	…………….	$\left(\varphi_P^{(m)}(u_k), \Psi_P^{(m)}(u_k)\right)$

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The matrix form of this tabular form is

$$\begin{pmatrix} \left(\varphi_P^{(1)}(u_1), \Psi_P^{(1)}(u_1)\right) & \left(\varphi_P^{(2)}(u_1), \Psi_P^{(2)}(u_1)\right) & \dots\dots\dots & \left(\varphi_P^{(m)}(u_1), \Psi_P^{(m)}(u_1)\right) \\ \left(\varphi_P^{(1)}(u_2), \Psi_P^{(1)}(u_2)\right) & \left(\varphi_P^{(2)}(u_2), \Psi_P^{(2)}(u_2)\right) & \dots\dots\dots & \left(\varphi_P^{(m)}(u_2), \Psi_P^{(m)}(u_2)\right) \\ \vdots & \vdots & & \vdots \\ \left(\varphi_P^{(1)}(u_k), \Psi_P^{(1)}(u_k)\right) & \left(\varphi_P^{(2)}(u_k), \Psi_P^{(2)}(u_k)\right) & \dots\dots\dots & \left(\varphi_P^{(m)}(u_k), \Psi_P^{(m)}(u_k)\right) \end{pmatrix}$$

Definition :2.6 [11]

Let 'm' be any positive integer. A Fermatean m-Polar fuzzy set (FmPFS) on U is defined as $F = \left\{\langle u, \left(\varphi_F^{(1)}(u), \Psi_F^{(1)}(u)\right), \left(\varphi_F^{(2)}(u), \Psi_F^{(2)}(u)\right), \dots, \left(\varphi_F^{(m)}(u), \Psi_F^{(m)}(u)\right)\rangle ; \forall\, u \in U\right\}$, where $\varphi_F^{(i)}: U \to [0,1]$ is the membership function $\Psi_F^{(i)}: U \to [0,1]$ is the non membership function which satisfies the condition $\left(\varphi_F^{(i)}(u)\right)^3 + \left(\Psi_F^{(i)}(u)\right)^3 \leq 1, \forall u \in U\,, i = 1,2,3, \dots, m$. The set $F = \left\{\langle u, \left(\varphi_F^{(1)}(u), \Psi_F^{(1)}(u)\right), \left(\varphi_F^{(2)}(u), \Psi_F^{(2)}(u)\right), \dots, \left(\varphi_F^{(m)}(u), \Psi_F^{(m)}(u)\right)\rangle ; \forall\, u \in U\right\}$ is rewrite as

$$F = \left\{\langle u, \left(\varphi_F^{(i)}(u), \Psi_F^{(i)}(u)\right)\rangle ; i = 1,2,3, \dots, m \text{ and } \forall\, u \in U\right\}$$

Table 2

U				
u_1	$\left(\varphi_F^{(1)}(u_1), \Psi_F^{(1)}(u_1)\right)$	$\left(\varphi_F^{(2)}(u_1), \Psi_F^{(2)}(u_1)\right)$	……..	$\left(\varphi_F^{(m)}(u_1), \Psi_F^{(m)}(u_1)\right)$
u_2	$\left(\varphi_F^{(1)}(u_2), \Psi_F^{(1)}(u_2)\right)$	$\left(\varphi_F^{(2)}(u_2), \Psi_F^{(2)}(u_2)\right)$	……..	$\left(\varphi_F^{(m)}(u_2), \Psi_F^{(m)}(u_2)\right)$
u_3	$\left(\varphi_F^{(1)}(u_3), \Psi_F^{(1)}(u_3)\right)$	$\left(\varphi_F^{(2)}(u_3), \Psi_F^{(2)}(u_3)\right)$	……..	$\left(\varphi_F^{(m)}(u_3), \Psi_F^{(m)}(u_3)\right)$
$\vdots$				
u_k	$\left(\varphi_F^{(1)}(u_k), \Psi_F^{(1)}(u_k)\right)$	$\left(\varphi_F^{(2)}(u_k), \Psi_F^{(2)}(u_k)\right)$	……..	$\left(\varphi_F^{(m)}(u_k), \Psi_F^{(m)}(u_k)\right)$

This table can be written in the matrix form

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$$\begin{pmatrix} \left(\varphi_F^{(1)}(u_1), \Psi_F^{(1)}(u_1)\right) & \left(\varphi_F^{(2)}(u_1), \Psi_F^{(2)}(u_1)\right) & \dots\dots\dots & \left(\varphi_F^{(m)}(u_1), \Psi_F^{(m)}(u_1)\right) \\ \left(\varphi_F^{(1)}(u_2), \Psi_F^{(1)}(u_2)\right) & \left(\varphi_F^{(2)}(u_2), \Psi_F^{(2)}(u_2)\right) & \dots\dots\dots & \left(\varphi_F^{(m)}(u_2), \Psi_F^{(m)}(u_2)\right) \\ \vdots & \vdots & & \vdots \\ \left(\varphi_F^{(1)}(u_k), \Psi_F^{(1)}(u_k)\right) & \left(\varphi_F^{(2)}(u_k), \Psi_F^{(2)}(u_k)\right) & \dots\dots\dots & \left(\varphi_F^{(m)}(u_k), \Psi_F^{(m)}(u_k)\right) \end{pmatrix}$$

The set contains all the Fermatean *m*-Polar fuzzy set is denoted by $\overline{FmPFS}$.

Let A be any F*m*PFS on U. Then $\pi_F^{(i)}(u) = \sqrt[3]{1 - \left(\varphi_F^{(i)}(u)\right)^3 - \left(\Psi_F^{(i)}(u)\right)^3}$, $i = 1,2, \dots, m$ is called the fermatean *m*-polar fuzzy set index or hesitation margin of u in U.The hesitation margin $\pi_F^{(i)}(u)$ is the degree of non-determinacy of u in U,to the set F and $\pi_F^{(i)}(u) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $u \in U$ or $u \notin U$. Thus

$$\left(\varphi_F^{(i)}(u)\right)^3 + \left(\Psi_F^{(i)}(u)\right)^3 + \left(\pi_F^{(i)}(u)\right)^3 = 1$$

Definition :2.7 [5]

Let $F(A \to B)$ and $G(B \to C)$ be two Intuitionistic fuzzy relations. Then, the max-min-max composite relation, $H = G \circ F$ is an Intuitionistic fuzzy relation from $A \to C$ is defined by

$$H = \{\langle (x, y), \varphi_F(x, y), \Psi_G(x, y) \rangle, x \in A \text{ and } y \in C\} ,$$

its membership and non-membership are defined by

$$\varphi_H(a, c) = \vee_b \{\wedge \left(\varphi_F(a, b), \varphi_G(b, c)\right)\}$$

$$\Psi_H(a, c) = \wedge_b \{\vee \left(\Psi_F(a, b), \Psi_G(b, c)\right)\}$$

$\forall (a, c) \in A \times C$ and $\forall b \in B$. Where $' \vee ' -$ maximum and $' \wedge ' -$ minimum

Definition :2.8 [6]

Let $F(A \to B)$ and $G(B \to C)$ be two Intuitionistic fuzzy relations. Then, the improved composite relation, $H = G \circ F$ is an Intuitionistic fuzzy relation from $A \to C$ is defined by

$$H = \{\langle (x, y), \varphi_F(x, y), \Psi_G(x, y) \rangle, x \in A \text{ and } y \in C\},$$

its membership and non-membership are defined by

$$\varphi_H(a, c) = \vee_b \{average\left(\varphi_F(a, b), \varphi_G(b, c)\right)\}$$

$$\Psi_H(a, c) = \wedge_b \{average\left(\Psi_F(a, b), \Psi_G(b, c)\right)\}$$

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$\forall(a,c) \in A \times C$ and $\forall b \in B$.

3.Classifications Of Composite Relations on Fermatean *m*-Polar Fuzzy Set

In this section we introduce the notion of Fermatean *m*-Polar fuzzy relations and some composite relations

Definition :3.1

Let U and V be two universal sets. Let A and B be two fermatean m-polar fuzzy sets on U and V respectively. Then Fermatean *m*-polar fuzzy relation $R = A \times B \subseteq U \times V$ defined by

$$R = \left\{\langle (x,y), \varphi_R^{(i)}(x,y), \Psi_R^{(i)}(x,y)\rangle, x \in U \ and \ y \in V, i = 1,2,\dots,m\right\}, \text{ where}$$

$$\varphi_R^{(i)}(x,y) = \min\left\{\varphi_A^{(i)}(x), \varphi_B^{(i)}(y)\right\}, i = 1,2,\dots,m$$

$$\Psi_R^{(i)}(x,y) = \max\left\{\Psi_A^{(i)}(x), \Psi_B^{(i)}(y)\right\}, i = 1,2,\dots,m$$

where $\varphi_R^{(i)}: U \times V \to [0,1]$ and $\Psi_R^{(i)}: U \times V \to [0,1]$ satisfies the condition

$$\left(\varphi_R^{(i)}(x,y)\right)^3 + \left(\Psi_R^{(i)}(x,y)\right)^3 \le 1, \forall (x,y) \in U \times V.$$

The relation R is denoted by $R(A \to B)$.

Definition :3.2

Suppose F and G are F*m*PFRs of $A \times B$ and $B \times C$ denoted by $F(A \to B)$ and $G(B \to C)$ respectively. Then Fermatean *m*-Polar fuzzy max-min-max composite relation (F*m*PFMMMCR), $H = G \circ F$ of $A \times C$ is of the form

$$H = \{\langle (a,c), \varphi_H(a,b), \Psi_H(\mathrm{b},\mathrm{c})\rangle / \ (a,c) \in A \times C\}, \text{ where}$$

$$\varphi_H^{(i)}(a,c) = \vee_b \left\{\wedge\left(\varphi_F^{(i)}(a,b), \varphi_G^{(i)}(b,c)\right), i = 1,2,\dots,m\right\}$$

$$\Psi_H^{(i)}(a,c) = \wedge_b \left\{\vee\left(\Psi_F^{(i)}(a,b), \Psi_G^{(i)}(b,c)\right), i = 1,2,\dots,m\right\}$$

$$\varphi_H(a,c) = \max\left\{\varphi_H^{(i)}(a,c), i = 1,2,\dots,m\right\} \ and \ \Psi_H(a,c) = \min\left\{\Psi_H^{(i)}(a,c), i = 1,2,\dots,m\right\}$$

such that $\left(\varphi_H^{(i)}(a,c)\right)^3 + \left(\Psi_H^{(i)}(a,c)\right)^3 \le 1$, $i = 1,2,\dots,m$ and

$$\pi_H(a,c) = \sqrt[3]{1 - \left(\varphi_H(a,c)\right)^3 - \left(\Psi_H(a,c)\right)^3}$$

The Fermatean *m*-Polar max min max composite relational value $H = G \circ F$ is calculated by

$H = \varphi_H(a,c) - \Psi_H(a,c)\pi_H(a,c)$, $\forall \ (a,c) \in A \times C$

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

Definition :3.3

Suppose F and G are F*m*PFRs of $A \times B$ and $B \times C$ denoted by $F(A \to B)$ and $G(B \to C)$ respectively. Then Improved fermatean *m*-polar fuzzy composite relation (IMF*m*PFCR),

$\overline{H} = G \circ F$ of $A \times C$ is of the form $\overline{H} = \{\langle (a,c), \varphi_{\overline{H}}(a,b), \Psi_{\overline{H}}(\mathrm{b,c})\rangle / (a,c) \in A \times C\}$, where

$$\varphi_{\overline{H}}^{(i)}(a,c) = \vee_b \left\{average\left(\varphi_F^{(i)}(a,b), \varphi_G^{(i)}(b,c)\right) / \, i = 1,2,\dots,m\right\}$$

$$\Psi_{\overline{H}}^{(i)}(a,c) = \wedge_b \left\{average\left(\Psi_F^{(i)}(a,b), \Psi_G^{(i)}(b,c)\right) / \, i = 1,2,\dots,m\right\}$$

$\Phi_{\overline{H}}(a,c) = \max\left\{\varphi_{\overline{H}}^{(i)}(a,c)\,, i = 1,2,\dots,m\right\}$ *and*

$\Psi_{\overline{H}}(a,c) = \min\left\{\Psi_{\overline{H}}^{(i)}(a,c), i = 1,2,\dots,m\right\}$, such that

$$\left(\varphi_{\overline{H}}^{(i)}(a,c)\right)^3 + \left(\Psi_{\overline{H}}^{(i)}(a,c)\right)^3 \le 1\,, i = 1,2,\dots,m \; and$$

$\pi_{\overline{H}}(a,c) = \sqrt[3]{1 - \left(\varphi_{\overline{H}}(a,c)\right)^3 - \left(\Psi_{\overline{H}}(a,c)\right)^3}$

The Improved Fermatean *m*-Polar fuzzy composite relational value $\overline{H} = G \circ F$ is calculated by

$\overline{H} = \varphi_{\overline{H}}(a,c) - \Psi_{\overline{H}}(a,c)\pi_{\overline{H}}(a,c)$, $\forall\, (a,c) \in A \times C$

Definition :3.4

Suppose F and G are F*m*PFRs of $A \times B$ and $B \times C$ denoted by $F(A \to B)$ and $G(B \to C)$ respectively. Then Refined fermatean *m*-polar fuzzy composite relation (RF*m*PFCR),

$\breve{H} = G \circ F$ of $A \times C$ is of the form

$$\breve{H}(a,b) = \{\langle (a,c), \varphi_{\breve{H}}(a,b), \Psi_{\breve{H}}(\mathrm{b,c})\rangle / (a,c) \in A \times C\}$$, where

$$\varphi_{\breve{H}}^{(i)}(a,c) = \vee_b \left\{\sqrt{(\varphi_F^{(i)}(a,b), \varphi_G^{(i)}(b,c))} \; / \, i = 1,2,\dots,m\right\}$$

$$\Psi_{\breve{H}}^{(i)}(a,c) = \wedge_b \left\{\sqrt{(\Psi_F^{(i)}(a,b), \Psi_G^{(i)}(b,c))} \; / \, i = 1,2,\dots,m\right\}$$

$\Phi_{\breve{H}}(a,c) = \max\left\{\varphi_{\breve{H}}^{(i)}(a,c)\,, i = 1,2,\dots,m\right\}$ *and*

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$\Psi_{\breve{H}}(a,c) = \min\left\{\Psi_{\breve{H}}^{(i)}(a,c), i = 1,2,\dots,m\right\}$, such that

$\left(\varphi_{\breve{H}}^{(i)}(a,c)\right)^3 + \left(\Psi_{\breve{H}}^{(i)}(a,c)\right)^3 \leq 1, i = 1,2,\dots,m \text{ and}$

$\pi_{\breve{H}}(a,c) = \sqrt[3]{1 - \left(\varphi_{\breve{H}}(a,c)\right)^3 - \left(\Psi_{\breve{H}}(a,c)\right)^3}$

The Refined Fermatean *m*-Polar fuzzy composite relational value $\breve{H} = G \circ F$ is calculated by

$\breve{H} = \varphi_{\breve{H}}(a,c) - \Psi_{\breve{H}}(a,c)\pi_{\breve{H}}(a,c)$, $\forall (a,c) \in A \times C$

Example :3.5

Let $A = \begin{matrix} x_1 \\ x_2 \end{matrix}\begin{pmatrix} (0.7,0.8) & (0.5,0.9) \\ (0.2,0.4) & (0.1,0.8) \end{pmatrix}$, $B = \begin{matrix} y_1 \\ y_2 \end{matrix}\begin{pmatrix} (0.4,0.2) & (0.9,0.1) \\ (0.8,0.6) & (0.2,0.7) \end{pmatrix}$ and

$C = \begin{matrix} z_1 \\ z_2 \end{matrix}\begin{pmatrix} (0.6,0.7) & (0.4,0.5) \\ (0.1,0.8) & (0.9,0.2) \end{pmatrix}$ be fermatean 2-polar fuzzy sets on the universal sets $U=\{x_1, x_2\}$ $V=\{y_1, y_2\}$ and $W=\{z_1, z_2\}$ respectively.

Now we find the relation $F = A \times B$

$\varphi_{AXB}^{(1)}(x_1, y_1) = \min\{\varphi_A^{(1)}(x_1), \varphi_B^{(1)}(y_1)\} = \min\{0.7,0.4\} = 0.4$

$\Psi_{AXB}^{(1)}(x_1, y_1) = \max\{\Psi_A^{(1)}(x_1), \Psi_B^{(1)}(y_1)\} = \max\{0.8,0.2\} = 0.8$

$\varphi_{AXB}^{(2)}(x_1, y_1) = \min\{\varphi_A^{(2)}(x_1), \varphi_B^{(2)}(y_1)\} = \min\{0.5,0.9\} = 0.5$

$\Psi_{AXB}^{(2)}(x_1, y_1) = \max\{\Psi_A^{(2)}(x_1), \Psi_B^{(2)}(y_1)\} = \max\{0.9,0.1\} = 0.9$

$\varphi_{AXB}^{(1)}(x_1, y_2) = \min\{\varphi_A^{(1)}(x_1), \varphi_B^{(1)}(y_2)\} = \min\{0.7,0.8\} = 0.7$

$\Psi_{AXB}^{(1)}(x_1, y_2) = \max\{\Psi_A^{(1)}(x_1), \Psi_B^{(1)}(y_2)\} = \max\{0.8,0.6\} = 0.8$

$\varphi_{AXB}^{(2)}(x_1, y_2) = \min\{\varphi_A^{(2)}(x_1), \varphi_B^{(2)}(y_2)\} = \min\{0.5,0.2\} = 0.2$

$\Psi_{AXB}^{(2)}(x_1, y_2) = \max\{\Psi_A^{(2)}(x_1), \Psi_B^{(2)}(y_2)\} = \max\{0.9,0.7\} = 0.9$

$\varphi_{AXB}^{(1)}(x_2, y_1) = \min\{\varphi_A^{(1)}(x_2), \varphi_B^{(1)}(y_1)\} = \min\{0.2,0.4\} = 0.2$

$\Psi_{AXB}^{(1)}(x_2, y_1) = \max\{\Psi_A^{(1)}(x_2), \Psi_B^{(1)}(y_1)\} = \max\{0.4,0.2\} = 0.4$

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$\varphi^{(2)}_{AXB}(x_2, y_1) = \min\{\varphi^{(2)}_{A}(x_2), \varphi^{(2)}_{B}(y_1)\} = \min\{0.1, 0.9\} = 0.1$

$\Psi^{(2)}_{AXB}(x_2, y_1) = \max\{\Psi^{(2)}_{A}(x_2), \Psi^{(2)}_{B}(y_1)\} = \max\{0.8, 0.1\} = 0.8$

$\varphi^{(1)}_{AXB}(x_2, y_2) = \min\{\varphi^{(1)}_{A}(x_2), \varphi^{(1)}_{B}(y_2)\} = \min\{0.2, 0.8\} = 0.2$

$\Psi^{(1)}_{AXB}(x_2, y_2) = \max\{\Psi^{(1)}_{A}(x_2), \Psi^{(1)}_{B}(y_2)\} = \max\{0.4, 0.6\} = 0.6$

$\varphi^{(2)}_{AXB}(x_2, y_2) = \min\{\varphi^{(2)}_{A}(x_2), \varphi^{(2)}_{B}(y_2)\} = \min\{0.1, 0.2\} = 0.1$

$\Psi^{(2)}_{AXB}(x_2, y_2) = \max\{\Psi^{(2)}_{A}(x_2), \Psi^{(2)}_{B}(y_2)\} = \max\{0.8, 0.7\} = 0.8$

Table 3: $F = A \times B$

$F = A \times B$	y_1	y_2
x_1	(0.4,0.8) (0.5,0.9)	(0.7,0.8) (0.2,0.9)
x_2	(0.2,0.4) (0.1,0.8)	(0.2,0.6) (0.1,0.8)

Similarly, we can find

Table 4: $G = B \times C$

$G = B \times C$	z_1	z_2
y_1	(0.4,0.7) (0.4,0.5)	(0.1,0.8) (0.9,0.2)
y_2	(0.6,0.7) (0.2,0.7)	(0.1,0.8) (0.2,0.7)

Here, $F(A \to B)$ and $G(B \to C)$. Let $H = G \circ F$

Now we find H

(i) using max-min-max composite relation

$$\varphi^{(1)}_{H}(x_1, z_1) = \vee\left\{\wedge\left(\varphi^{(1)}_{F}(x_1, y_1), \varphi^{(1)}_{G}(y_1, z_1)\right), \wedge\left(\varphi^{(1)}_{F}(x_1, y_2), \varphi^{(1)}_{G}(y_2, z_1)\right)\right\}$$

$$= \vee\{\wedge(0.4, 0.4), \wedge(0.7, 0.6)\} = \vee\{0.4, 0.6\} = 0.6$$

$$\Psi^{(1)}_{H}(x_1, z_1) = \wedge\left\{\vee\left(\Psi^{(1)}_{F}(x_1, y_1), \Psi^{(1)}_{G}(y_1, z_1)\right), \vee\left(\Psi^{(1)}_{F}(x_1, y_2), \Psi^{(1)}_{G}(y_2, z_1)\right)\right\}$$

$$= \wedge\{\vee(0.8, 0.7), \vee(0.8, 0.7)\} = \wedge\{0.8, 0.8\} = 0.8$$

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$$\varphi_H^{(2)}(x_1, z_1) = \vee\left\{\wedge\left(\varphi_F^{(2)}(x_1, y_1), \varphi_G^{(2)}(y_1, z_1)\right), \wedge\left(\varphi_F^{(2)}(x_1, y_2), \varphi_G^{(2)}(y_2, z_1)\right)\right\}$$
$$= \vee\{\wedge(0.5,0.4), \wedge(0.2,0.2)\} = \vee\{0.4,0.2\} = 0.4$$

$$\Psi_H^{(2)}(x_1, z_1) = \wedge\left\{\vee\left(\Psi_F^{(2)}(x_1, y_1), \Psi_G^{(2)}(y_1, z_1)\right), \vee\left(\Psi_F^{(2)}(x_1, y_2), \Psi_G^{(2)}(y_2, z_1)\right)\right\}$$
$$= \wedge\{\vee(0.9,0.5), \vee(0.9,0.7)\} = \wedge\{0.9,0.9\} = 0.9$$

$$\varphi_H^{(1)}(x_1, z_2) = \vee\left\{\wedge\left(\varphi_F^{(1)}(x_1, y_1), \varphi_G^{(1)}(y_1, z_2)\right), \wedge\left(\varphi_F^{(1)}(x_1, y_2), \varphi_G^{(1)}(y_2, z_2)\right)\right\}$$
$$= \vee\{\wedge(0.4,0.1), \wedge(0.7,0.1)\} = \vee\{0.1,0.1\} = 0.1$$

$$\Psi_H^{(1)}(x_1, z_2) = \wedge\left\{\vee\left(\Psi_F^{(1)}(x_1, y_1), \Psi_G^{(1)}(y_1, z_2)\right), \wedge\left(\Psi_F^{(1)}(x_1, y_2), \Psi_G^{(1)}(y_2, z_2)\right)\right\}$$
$$= \wedge\{\vee(0.8,0.8), \wedge(0.8,0.8)\} = \wedge\{0.8,0.8\} = 0.8$$

$$\varphi_H^{(2)}(x_1, z_2) = \vee\left\{\wedge\left(\varphi_F^{(2)}(x_1, y_1), \varphi_G^{(2)}(y_1, z_2)\right), \wedge\left(\varphi_F^{(2)}(x_1, y_2), \varphi_G^{(2)}(y_2, z_2)\right)\right\}$$
$$= \vee\{\wedge(0.5,0.9), \wedge(0.2,0.2)\} = \vee\{0.5,0.2\} = 0.5$$

$$\Psi_H^{(2)}(x_1, z_2) = \wedge\left\{\vee\left(\Psi_F^{(2)}(x_1, y_1), \Psi_G^{(2)}(y_1, z_2)\right), \vee\left(\Psi_F^{(2)}(x_1, y_2), \Psi_G^{(2)}(y_2, z_2)\right)\right\}$$
$$= \wedge\{\vee(0.9,0.2), \vee(0.9,0.7)\} = \wedge\{0.9,0.9\} = 0.9$$

$$\varphi_H^{(1)}(x_2, z_1) = \vee\left\{\wedge\left(\varphi_F^{(1)}(x_2, y_1), \varphi_G^{(1)}(y_1, z_1)\right), \wedge\left(\varphi_F^{(1)}(x_2, y_2), \varphi_G^{(1)}(y_2, z_1)\right)\right\}$$
$$= \vee\{\wedge(0.2,0.4), \wedge(0.2,0.6)\} = \vee\{0.2,0.2\} = 0.2$$
$$\Psi_H^{(1)}(x_2, z_1) = \wedge\left\{\vee\left(\Psi_F^{(1)}(x_2, y_1), \Psi_G^{(1)}(y_1, z_1)\right), \vee\left(\Psi_F^{(1)}(x_2, y_2), \Psi_G^{(1)}(y_2, z_1)\right)\right\}$$
$$= \wedge\{\vee(0.4,0.7), \vee(0.6,0.7)\} = \wedge\{0.7,0.7\} = 0.7$$

$$\varphi_H^{(2)}(x_2, z_1) = \vee\left\{\wedge\left(\varphi_F^{(2)}(x_2, y_1), \varphi_G^{(2)}(y_1, z_1)\right), \wedge\left(\varphi_F^{(2)}(x_2, y_2), \varphi_G^{(2)}(y_2, z_1)\right)\right\}$$
$$= \vee\{\wedge(0.1,0.4), \wedge(0.1,0.2)\} = \vee\{0.1,0.1\} = 0.1$$

$$\Psi_H^{(2)}(x_2, z_1) = \wedge\left\{\vee\left(\Psi_F^{(2)}(x_2, y_1), \Psi_G^{(2)}(y_1, z_1)\right), \vee\left(\Psi_F^{(2)}(x_2, y_2), \Psi_G^{(2)}(y_2, z_1)\right)\right\}$$
$$= \wedge\{\vee(0.8,0.5), \vee(0.8,0.7)\} = \wedge\{0.8,0.8\} = 0.8$$

RCHUB: JOURNAL OF COMPUTATIONAL INTELLIGENCE SCIENCE AND ENGINEERING (JCISE)
VOL: 2, ISSUE 1, JANUARY-MARCH 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$$\varphi_H^{(1)}(x_2, z_2) = \vee \left\{ \wedge \left(\varphi_F^{(1)}(x_2, y_1), \varphi_G^{(1)}(y_1, z_2) \right), \wedge \left(\varphi_F^{(1)}(x_2, y_2), \varphi_G^{(1)}(y_2, z_2) \right) \right\}$$
$$= \vee \{\wedge(0.2,0.1), \wedge(0.2,0.1)\} = \vee \{0.1,0.1\} = 0.1$$

$$\Psi_H^{(1)}(x_2, z_2) = \wedge \left\{ \vee \left(\Psi_F^{(1)}(x_2, y_1), \Psi_G^{(1)}(y_1, z_2) \right), \vee \left(\Psi_F^{(1)}(x_2, y_2), \Psi_G^{(1)}(y_2, z_2) \right) \right\}$$
$$= \wedge \{\vee(0.4,0.8), \vee(0.6,0.8)\} = \wedge \{0.8,0.8\} = 0.8$$

$$\varphi_H^{(2)}(x_2, z_2) = \vee \left\{ \wedge \left(\varphi_F^{(2)}(x_2, y_1), \varphi_G^{(2)}(y_1, z_2) \right), \wedge \left(\varphi_F^{(2)}(x_2, y_2), \varphi_G^{(2)}(y_2, z_2) \right) \right\}$$
$$= \vee \{\wedge(0.1,0.9), \wedge(0.1,0.2)\} = \vee \{0.1,0.1\} = 0.1$$

$$\Psi_H^{(2)}(x_2, z_2) = \wedge \left\{ \vee \left(\Psi_F^{(2)}(x_2, y_1), \Psi_G^{(2)}(y_1, z_2) \right), \vee \left(\Psi_F^{(2)}(x_2, y_2), \Psi_G^{(2)}(y_2, z_2) \right) \right\}$$
$$= \wedge \{\vee(0.8,0.2), \vee(0.8,0.7)\} = \wedge \{0.8,0.8\} = 0.8$$

Table 5: $H = G \circ F$

H	z_1	z_2
x_1	(0.6,0.8) (0.4,0.9)	(0.1,0.8) (0.5,0.9)
x_2	(0.2,0.7) (0.1,0.8)	(0.1,0.8) (0.1,0.8)

(ii) using improved composite relation $\overline{H} = G \circ F$

$\varphi_{\overline{H}}^{(1)}(x_1, z_1) = \vee \{\text{average}(0.4,0.4), \text{average}(0.7,0.6)\} = \vee \{0.4,0.65\} = 0.65$

$\Psi_{\overline{H}}^{(1)}(x_1, z_1) = \wedge \{\text{average}(0.8,0.7), \text{average}(0.8,0.7)\} = \wedge \{0.75,0.75\} = 0.75$

$\varphi_{\overline{H}}^{(2)}(x_1, z_1) = \vee \{\text{average}(0.5,0.4), \text{average}(0.2,0.2)\} = \vee \{0.45,0.2\} = 0.45$

$\Psi_{\overline{H}}^{(2)}(x_1, z_1) = \wedge \{\text{average}(0.9,0.5), \text{average}(0.9,0.7)\} = \wedge \{0.7,0.8\} = 0.7$

$\varphi_{\overline{H}}^{(1)}(x_1, z_2) = \vee \{\text{average}(0.4,0.1), \text{average}(0.7,0.1)\} = \vee \{0.25,0.4\} = 0.4$

$\Psi_{\overline{H}}^{(1)}(x_1, z_2) = \wedge \{\text{average}(0.8,0.8), \text{average}(0.8,0.8)\} = \wedge \{0.8,0.8\} = 0.8$

RCHUB: JOURNAL OF COMPUTATIONAL INTELLIGENCE SCIENCE AND ENGINEERING (JCISE)
VOL: 2, ISSUE 1, JANUARY-MARCH 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$\varphi_{\bar{H}}^{(2)}(x_1, z_2) = \vee \{\text{average}(0.5,0.9), \text{average}(0.2,0.2)\} = \vee \{0.7,0.2\} = 0.7$

$\Psi_{\bar{H}}^{(2)}(x_1, z_2) = \wedge \{\text{average}(0.9,0.2), \text{average}(0.9,0.7)\} = \wedge \{0.55,0.8\} = 0.55$

$\varphi_{\bar{H}}^{(1)}(x_2, z_1) = \vee \{\text{average}(0.2,0.4), \text{average}(0.2,0.6)\} = \vee \{0.3,0.4\} = 0.4$

$\Psi_{\bar{H}}^{(1)}(x_2, z_1) = \wedge \{\text{average}(0.4,0.7), \text{average}(0.6,0.7)\} = \wedge \{0.55,0.65\} = 0.55$

$\varphi_{\bar{H}}^{(2)}(x_2, z_1) = \vee \{\text{average}(0.1,0.4), \text{average}(0.1,0.2)\} = \vee \{0.25,0.15\} = 0.25$

$\Psi_{\bar{H}}^{(2)}(x_2, z_1) = \wedge \{\text{average}(0.8,0.5), \text{average}(0.8,0.7)\} = \wedge \{0.65,0.75\} = 0.65$

$\varphi_{\bar{H}}^{(1)}(x_2, z_2) = \vee \{\text{average}(0.2,0.1), \text{average}(0.2,0.1)\} = \vee \{0.15,0.15\} = 0.15$

$\Psi_{\bar{H}}^{(1)}(x_2, z_2) = \wedge \{\text{average}(0.4,0.8), \text{average}(0.6,0.8)\} = \wedge \{0.6,0.7\} = 0.6$

$\varphi_{\bar{H}}^{(2)}(x_2, z_2) = \vee \{\text{average}(0.1,0.9), \text{average}(0.1,0.2)\} = \vee \{0.5,0.15\} = 0.5$

$\Psi_{\bar{H}}^{(2)}(x_2, z_2) = \wedge \{\text{average}(0.8,0.2), \text{average}(0.8,0.7)\} = \wedge \{0.5,0.75\} = 0.5$

Table 6: $\bar{H} = G \circ F$

$\bar{H}$	z_1	z_2
x_1	(0.65,0.75) (0.45,0.7)	(0.4,0.8) (0.7,0.55)
x_2	(0.4,0.55) (0.25,0.65)	(0.15,0.6) (0.5,0.5)

(iii) using refined composite relation

$\varphi_{\breve{H}}^{(1)}(x_1, z_1) == \vee \{\sqrt{0.4 \times 0.4}, \sqrt{0.7 \times 0.6}\} = \vee \{0.4,0.6480\} = 0.6480$

$\Psi_{\breve{H}}^{(1)}(x_1, z_1) = \wedge \{\sqrt{0.8 \times 0.7}, \sqrt{0.8 \times 0.7}\} = \wedge \{0.7483,0.7483\} = 0.7483$

$\varphi_{\breve{H}}^{(2)}(x_1, z_1) = \vee \{\sqrt{0.5 \times 0.4}, \sqrt{0.2 \times 0.2}\} = \vee \{0.4472,0.2\} = 0.4472$

$\Psi_{\breve{H}}^{(2)}(x_1, z_1) = \wedge \{\sqrt{0.9 \times 0.5}, \sqrt{0.9 \times 0.7}\} = \wedge \{0.6708,0.7937\} = 0.6708$

$\varphi_{\breve{H}}^{(1)}(x_1, z_2) = \vee \{\sqrt{0.4 \times 0.1}, \sqrt{0.7 \times 0.1}\} = \vee \{0.2,0.2645\} = 0.2645$

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

$$\Psi_{\breve{H}}^{(1)}(x_1, z_2) = \wedge\left\{\sqrt{0.8 \times 0.8}, \sqrt{0.8 \times 0.8}\right\} = \wedge\{0.8, 0.8\} = 0.8$$

$$\varphi_{\breve{H}}^{(2)}(x_1, z_2) = \vee\left\{\sqrt{0.5 \times 0.9}, \sqrt{0.2 \times 0.2}\right\} = \vee\{0.6708, 0.2\} = 0.6708$$

$$\Psi_{\breve{H}}^{(2)}(x_1, z_2) = \wedge\left\{\sqrt{0.9 \times 0.2}, \sqrt{0.9 \times 0.7}\right\} = \wedge\{0.4242, 0.7937\} = 0.4242$$

$$\varphi_{\breve{H}}^{(1)}(x_2, z_1) = \vee\left\{\sqrt{0.2 \times 0.4}, \sqrt{0.2 \times 0.6}\right\} = \vee\{0.2828, 0.3464\} = 0.3464$$

$$\Psi_{\breve{H}}^{(1)}(x_2, z_1) = \wedge\left\{\sqrt{0.4 \times 0.7}, \sqrt{0.6 \times 0.7}\right\} = \wedge\{0.5291, 0.6480\} = 0.5291$$

$$\varphi_{\breve{H}}^{(2)}(x_2, z_1) = \vee\left\{\sqrt{0.1 \times 0.4}, \sqrt{0.1 \times 0.2}\right\} = \vee\{0.2, 0.1414\} = 0.2$$

$$\Psi_{\breve{H}}^{(2)}(x_2, z_1) = \wedge\left\{\sqrt{0.8 \times 0.5}, \sqrt{0.8 \times 0.7}\right\} = \wedge\{0.6324, 0.7483\} = 0.6324$$

$$\varphi_{\breve{H}}^{(1)}(x_2, z_2) = \vee\left\{\sqrt{0.2 \times 0.1}, \sqrt{0.2 \times 0.1}\right\} = \vee\{0.1414, 0.1414\} = 0.1414$$

$$\Psi_{\breve{H}}^{(1)}(x_2, z_2) = \wedge\left\{\sqrt{0.4 \times 0.8}, \sqrt{0.6 \times 0.8}\right\} = \wedge\{0.5656, 0.6928\} = 0.5656$$

$$\varphi_{\breve{H}}^{(2)}(x_2, z_2) = \vee\left\{\sqrt{0.1 \times 0.9}, \sqrt{0.1 \times 0.2}\right\} = \vee\{0.3, 0.1414\} = 0.1414$$

$$\Psi_{\breve{H}}^{(2)}(x_2, z_2) = \wedge\left\{\sqrt{0.8 \times 0.2}, \sqrt{0.8 \times 0.7}\right\} = \wedge\{0.4, 0.7483\} = 0.4$$

Table 7: $\breve{H} = G \circ F$

$\breve{H}$	z_1	z_2
x_1	(0.6480,0.7483) (0.4472,0.6708)	(0.2645,0.8) (0.6708,0.4242)
x_2	(0.3464,0.5291) (0.2,0.6324)	(0.1414,0.5656) (0.1414,0.4)

3.6. Application of the improved composite relation for Fermatean *m*-Polar Fuzzy Sets in Medical Diagnosis

Consider four patients $P_1, P_{2,} P_{3,} P_4$ who arrive at a hospital presenting the symptoms temperature, caugh, breathing difficulty, throat pain, and loss of taste.

Thus, the set of patients is defined as $P = \{P_1, P_2, P_3, P_4\}$ and the symptom set is

$$Q = \{Temperature, Cough, Throat\ pain, Breathing\ difficulty, Loss\ of\ taste\}.$$

RCHUB: JOURNAL OF COMPUTATIONAL INTELLIGENCE SCIENCE AND ENGINEERING (JCISE)
VOL: 2, ISSUE 1, JANUARY-MARCH 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

Clinical observations are recorded over three instances: Day 1, Day 2, Day 3. The fermatean 3-polar fuzzy relation $R_1(P \to Q)$ corresponding to these observations is hypothetically represented in **Table 8**.

Let the set of possible diseases $D = \{Viral\ fever, Tuberculosis, Asthma, Covid\}$. The fermatean 3-polar fuzzy relation $R_2(Q \to D)$ is also defined hypothetically and presented in **Table 9**.

Table 8: $R_1(P \to Q)$

R_1	$Temperature$	$Cough$	$Throat\ pain$	$Breathing\ difficulty$	$Loss\ of\ taste$
P_1	(0.6,0.5) (0.4,0.6) (0.2,0.7)	(0.8,0.2) (0.7,0.3) (0.9,0.4)	(0.4,0.6) (0.2,0.5) (0.7,0.1)	(0.5,0.7) (0.4,0.3) (0.2,0.1)	(0.3,0.4) (0.8,0.5) (0.6,0.3)
P_2	(0.9,0.2) (0.8,0.4) (0.7,0.2)	(0.5,0.6) (0.4,0.3) (0.7,0.5)	(0.2,0.5) (0.6,0.4) (0.5,0.3)	(0.7,0.4) (0.5,0.3) (0.2,0.4)	(0.1,0.6) (0.8,0.5) (0.3,0.7)
P_3	(0.6,0.3) (0.5,0.6) (0.7,0.4)	(0.3,0.5) (0.6,0.4) (0.2,0.3)	(0.5,0.4) (0.6,0.3) (0.7,0.5)	(0.7,0.4) (0.8,0.3) (0.5,0.2)	(0.9,0.4) (0.7,0.1) (0.6,0.3)
P_4	(0.5,0.2) (0.4,0.7) (0.8,0.3)	(0.2,0.4) (0.7,0.3) (0.6,0.1)	(0.4,0.7) (0.6,0.2) (0.5,0.1)	(0.8,0.2) (0.7,0.3) (0.9,0.4)	(0.5,0.3) (0.2,0.6) (0.3,0.1)

Table 9: $R_2(Q \to D)$

R_2	$Viral\ fever$	$Tuberculosis$	$Asthma$	$Covid$
$Temperature$	(0.8,0.1) (0.6,0.3) (0.5,0.2)	(0.5,0.6) (0.3,0.7) (0.6,0.4)	(0.6,0.3) (0.3,0.1) (0.5,0.4)	(0.4,0.5) (0.3,0.2) (0.2,0.3)
$Cough$	(0.7,0.5)	(0.9,0.3)	(0.4,0.6)	(0.1,0.6)

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

	(0.2,0.7) (0.1,0.4)	(0.8,0.7) (0.7,0.6)	(0.3,0.4) (0.2,0.5)	(0.5,0.3) (0.4,0.7)
$Throat\ pain$	(0.5,0.4) (0.4,0.3) (0.6,0.7)	(0.2,0.4) (0.6,0.2) (0.5,0.1)	(0.7,0.3) (0.4,0.6) (0.7.0.8)	(0.7,0.2) (0.8,0.5) (0.6,0.7)
$Breathing\ difficulty$	(0.4,0.1) (0.6,0.7) (0.1,0.5)	(0.1,0.3) (0.6,0.4) (0.3,0.5)	(0.9,0.3) (0.8,0.2) (0.6,0.2)	(0.6,0.3) (0.7,0.5) (0.8,0.4)
$Loss\ of\ taste$	(0.2,0.4) (0.5,0.6) (0.4,0.5)	(0.5,0.6) (0.7,0.5) (0.2,0.4)	(0.4,0.6) (0.7,0.5) (0.5,0.3)	(0.8,0.3) (0.5,0.2) (0.7,0.4)

Table 10: $\varphi_R(p,d)\ and\ \Psi_R(p,d)$

R	$Viral\ fever$	$Tuberculosis$	$Asthma$	$Covid$
P_1	(0.75,0.3)	(0.85,0.1)	(0.75,0.15)	(0.65,0.25)
P_2	(0.85,0.15)	(0.75,0.2)	(0.8,0.25)	(0.7,0.25)
P_3	(0.7,0.2)	(0.7,0.25)	(0.8,0.2)	(0.85,0.15)
P_4	(0.65,0.15)	(0.75,0.1)	(0.85,0.2)	(0.85,0.25)

Table 11: $\varphi_R(p,d) - \Psi_R(p,d)\pi_R(p,d)$

R	$Viral\ fever$	$Tuberculosis$	$Asthma$	$Covid$
P_1	0.5040	**0.7772**	0.6252	0.4269
P_2	**0.7411**	0.5841	0.6052	0.4844
P_3	0.5268	0.4844	0.6434	**0.7411**
P_4	0.5154	0.6667	**0.7054**	0.6704

According to the findings presented in **Table 11**, Patient 1 is predicted to have Tuberculosis, Patient 2 is likely to develop Viral fever, is expected to be affected by Covid, and Patient 4 is projected to suffer from Asthma.

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

3.7 Application of the refined composite relation for Fermatean m-Polar Fuzzy Sets in Decision-Making

Consider three customers

$$A = \{Mani, Dhesh, Varni\}$$

who visit a car showroom with specific expectations regarding vehicle performance in highway and urban driving conditions. Their evaluation criteria are represented as

$$B = \{Mileage\ , Comfort, Safety\}.$$

The fermatean 2-polar fuzzy relation $R_1(A \to B)$ is hypothetically defined in **Table 12**, expressing the degree to which each customer associates with the given criteria. Let the available car models be

$$C = \{Model\ X, Model\ Y, Model\ Z\}$$

The fermatean 2-polar fuzzy relation $R_2(B \to C)$ is hypothetically presented in **Table 13**, indicating how strongly each vehicle model satisfies the specified criteria.

Table 12: $R_1(A \to B)$

R_1	$Mileage$	$Comfort$	$Safety$
$Mani$	(0.8,0.4) (0.6,0.5)	(0.5,0.3) (0.6,.0.5)	(0.7,0.6) (0.4,0.8)
$Dhesh$	(0.4,0.5) (0.3,0.1)	(0.9.0.6) (0.7.0.4)	(0.6,0.5) (0.7,0.8)
$Varni$	(0.5,0.2) (0.6,0.7)	(0.3,0.1) (0.4,0.2)	(0.9,0.2) (0.8,0.3)

Table 13: $R_2(B \to C)$

R_2	$Model\ X$	$Model\ Y$	$Model\ Z$
$Mileage$	(0.6,0.5) (0.4,0.7)	(0.9,0.3) (0.5,0.4)	(0.4,0.6) (0.2,0.3)
$Comfort$	(0.3,0.2) (0.5,0.7)	(0.7,0.1) (0.4,0.2)	(0.8,0.3) (0.7.0.5)
$Safety$	(0.8,0.5) (0.6,0.7)	(0.3,0.6) (0.5,0.2)	(0.6,0.5) (0.1,0.4)

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RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

Table 14: $R(A \to C)$

R	$Model\ X$	$Model\ Y$	$Model\ Z$
$Mani$	(0.7483,0.2449)	(0.8485,0.1732)	(0.6480,0.3)
$Dhesh$	(0.6928,0.2645)	(0.7937,0.2)	(0.8485,0.1732)
$Varni$	(0.8485,0.1414)	(0.6708,0.1)	(0.7348,0.1732)

Table 15: *Relational value of* $R = (\varphi_R - \Psi_R \pi_R)$

Relational value of R	$Model\ X$	$Model\ Y$	$Model\ Z$
$Mani$	0.5640	**0.7411**	0.3815
$Dhesh$	0.4638	0.6358	**0.7226**
$Varni$	**0.7475**	0.5821	0.5888

From **Table 15**, it is concluded that Mani is most likely to purchase Model Y, Dhesh is expected to choose Model Z, and Varni is inclined to buy Model X.

3.8 Comparison of Max–Min–Max, Improved, and Refined Composite Relations in Fermatean *m*-Polar Fuzzy Sets

Consider three voters $V = \{V_1, V_2, V_3\}$ who arrive at a polling station with expectations based on the criteria $E = \{Trustworthiness, Leadership, Policy\ Views\}$. The fermatean 3-polar fuzzy relation $R_1(V \to E)$ is hypothetically presented in **Table 16**, representing the degree to which each voter evaluates these criteria under different situations.

Let the set of political parties be $P = \{Party\ 1, Party\ 2, Party\ 3\}$. The fermatean 3-polar fuzzy relation $R_2(E \to P)$ is hypothetically defined in **Table 17**, indicating how well each party satisfies the specified criteria across various scenarios.

Table 16: $R_1(V \to E)$

R_1	$Trustworthiness$	$Leadership$	$Policy\ Views$
V_1	(0.3,0.5) (0.7,0.2) (0.4,0.8)	(0.2,0.5) (0.8,0.6) (0.7,0.4)	(0.8,0.7) (0.6,0.4) (0.3,0.5)
V_2	(0.1,0.7) (0.6,0.8) (0.5,0.3)	(0.6,0.3) (0.5,0.2) (0.1,0.7)	(0.3,0.4) (0.7,0.5) (0.2,0.6)
V_3	(0.4,0.2) (0.2,0.5) (0.7,0.1)	(0.1,0.5) (0.3,0.7) (0.5,0.8)	(0.5,0.4) (0.2,0.4) (0.3,0.7)

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Table 17: $R_2(E \to P)$

R_2	$Party\ 1$	$Party\ 2$	$Party\ 3$
$Trustworthiness$	(0.6,0.2) (0.5,0.4) (0.7,0.3)	(0.2,0.5) (0.3,0.4) (0.6,0.8)	(0.8,0.7) (0.5,0.4) (0.3.0.5)
$Leadership$	(0.4,0.1) (0.2,0.6) (0.8,0.5)	(0.6,0.7) (0.5,0.1) (0.4,0.2)	(0.5,0.4) (0.8,0.2) (0.1,0.6)
$Policy\ Views$	(0.3,0.5) (0.1,0.7) (0.4,0.2)	(0.8,0.6) (0.2,0.5) (0.7,0.1)	(0.6,0.2) (0.3,0.4) (0.5,0.7)

Table 18: $R(V \to P)$

$(\varphi_R(v,p)\ and\ \Psi_R(v,p))$ (max − min − max composite relation)

R	$Party\ 1$	$Party\ 2$	$Party\ 3$
V_1	(0.7,0.4)	(0.8,0.4)	(0.8,0.4)
V_2	(0.5,0.3)	(0.6,0.2)	(0.5,0.2)
V_3	(0.7,0.2)	(0.6,0.5)	(0.5,0.4)

Table 19: $\bar{R}(V \to P)$

($\varphi_{\bar{R}}(v,p)\ and\ \Psi_{\bar{R}}(v,p)$**)** (improved composite relation)

$\bar{R}$	$Party\ 1$	$Party\ 2$	$Party\ 3$
V_1	(0.75,0.3)	(0. 8,0.3)	(0. 8,0.3)
V_2	(0.6,0.2)	(0.6,0.15)	(0.6,0.2)
V_3	(0.7,0.2)	(0.65,0.35)	(0.6,0.3)

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Table 20: $\breve{R}(V \to P)$

($\varphi_{\breve{R}}(v,p)$ and $\Psi_{\breve{R}}(v,p)$) (refined composite relation)

$\breve{R}$	$Party\ 1$	$Party\ 2$	$Party\ 3$
V_1	(0.7483,0.2236)	(0.8 ,0.2236)	(0.8,0.2528)
V_2	(0.5916,0.1732)	(0.6,0.1414)	(0.5916,0.2)
V_3	(0.7,0.1732)	(0.6480,0.2645)	(0.5656,0.2236)

Table 21: $Relational\ value\ of\ \ R = (\varphi_R - \Psi_R \pi_R)$

$Relational\ value\ of\ \ R$	$Party\ 1$	$Party\ 2$	$Party\ 3$
V_1	0.3969	**0.4994**	**0.4994**
V_2	0.2160	**0.4162**	0.3092
V_3	**0.5268**	0.1648	0.1269

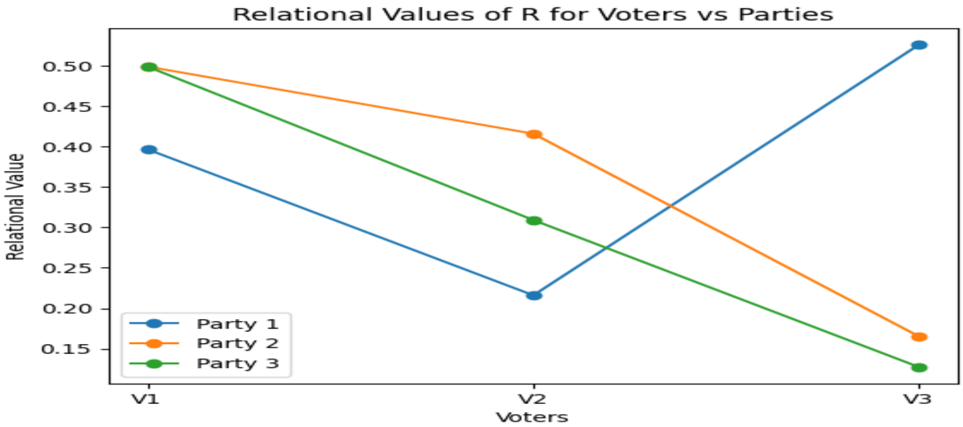

Figure 1: *Max-Min-Max composite relation*

Table 22: $Relational\ value\ of\ \ \bar{R} = (\varphi_{\bar{R}} - \Psi_{\bar{R}} \pi_{\bar{R}})$

$Relational\ value\ of\ \ \bar{R}$	$Party\ 1$	$Party\ 2$	$Party\ 3$
V_1	0.5040	**0.5682**	**0.5682**
V_2	0.4162	**0.4618**	0.4162
V_3	**0.5268**	0.3418	0.3265

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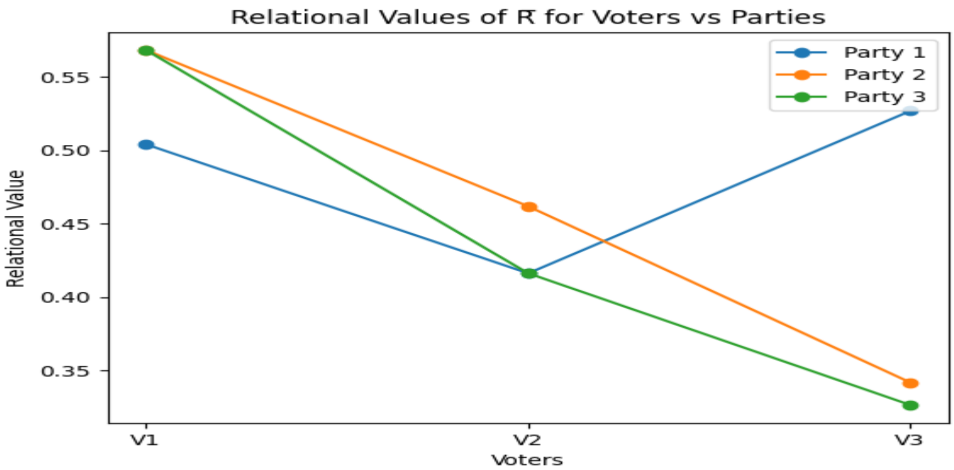

Figure 2: *Improved composite relation*

Table 23: $Relational\ value\ of\ \breve{R}\ = (\varphi_{\breve{R}} - \Psi_{\breve{R}}\pi_{\breve{R}})$

$Relational\ value\ of\ \breve{R}$	$Party\ 1$	$Party\ 2$	$Party\ 3$
V_1	0.5629	**0.6253**	0.6031
V_2	0.4316	**0.4697**	0.4071
V_3	**0.5498**	0.4121	0.3573

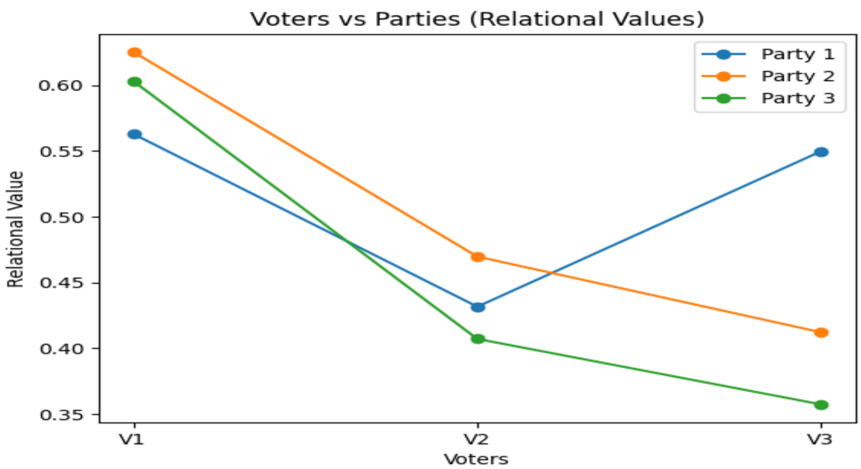

Figure 3: *Refined composite relation*

Table 21 gives the relational values using max-min-max composite relation, from this we

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

conclude that $(V_1, Party\ 2)$ and $(V_1, Party\ 3)$ have the same relational values. V_1 has decided to vote $Party\ 2$ and $Party\ 3$; V_2 has decided to vote $Party\ 2$; V_3 has decided to vote $Party\ 1$.

Table 22 gives the relational values using improved composite relation, from this we conclude that $(V_1, Party\ 2)$ and $(V_1, Party\ 3)$ have the same relational values. V_1 has decided to vote $Party\ 2$ and $Party\ 3$; V_2 has decided to vote $Party\ 2$; V_3 has decided to vote $Party\ 1$. $Party\ 2$

Table 23 gives the relational values using refined composite relation, from this we conclude that V_1 has decided to vote $Party\ 2$; V_2 has decided to vote $Party\ 2$; V_3 has decided to vote $Party\ 2$.

Hence, the refined fermatean *m*-polar fuzzy composite relation demonstrates superior performance compared with the other two composite relations. Moreover, the relational values obtained from the refined fermatean *m*-polar fuzzy composite model are higher than those derived using both the max-min-max fermatean *m*-polar fuzzy composite approach and the improved fermatean *m*-polar fuzzy composite method.

4.Conclusion

This work investigated relational structures defined on Fermatean *m*-Polar Fuzzy Sets and analysed three distinct composite mechanisms: the Max–Min–Max composite relation, the Improved composite relation, and the Refined composite relation. Each model was examined from both theoretical and application-oriented perspectives to evaluate its effectiveness in handling multi-polar uncertainty under the Fermatean condition.

The Max–Min–Max composition serves as a fundamental approach, ensuring logical consistency in relational mapping between sets. However, its aggregation strategy may limit the strength of relational propagation in complex decision environments. The Improved composite relation addresses this limitation by introducing a more adaptive composition process, enhancing the clarity and stability of relational outcomes, particularly in diagnostic modelling.

The Refined composite relation further strengthens the relational framework by optimizing the interaction among membership and non-membership grades across multiple poles. This refinement leads to better information retention during composition and produces more

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/1/014

discriminative relational values.

Applications in medical diagnosis and decision-making validate the practical relevance of the proposed models. While the Improved composite relation performs effectively in clinical evaluation scenarios, the Refined composite relation demonstrates stronger analytical capability in ranking and selection problems.

Comparative results clearly indicate that the Refined Fermatean *m*-Polar Fuzzy composite relation outperforms both the Max–Min–Max and Improved composite approaches. It consistently generates higher and more representative relational values, reflecting enhanced sensitivity and improved decision support capability. Therefore, the refined model can be considered the most robust and reliable composite framework among the three for addressing complex problems involving uncertainty and multi-criteria analysis.

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