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COMPUTING SOME DISTANCE - BASED TOPOLOGICAL INDICES OF THE CLUSTER CORONA PRODUCT OF K_M AND C_N

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Abstract

Distance-based topological indices are graph invariants that characterize graph structures using vertex distances. In this paper, we derive closed-form expressions for several distance-based topological indices of the cluster product of complete graphs and cycle graphs. The obtained results contribute to the theoretical study of graph products and provide a basis for further investigations of other graph classes and distance-based descriptors.

MSC: 05C76, 05C10, 05C38

Keywords: Wiener index, Hyper - Wiener Index, Harary Index, Reciprocal Complementary Wiener Index, Sum Connectivity Index, Batschelet Index, Reverse Wiener Index and Reciprocal Reverse Wiener Index, Wiener Polarity index, Eccentricity index and Cluster Product.

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1. Introduction

Distance based topological indices are numerical parameters that are computed using the shortest path distances between pairs of vertices in a graph. These indices capture the structural characteristics of a graph by quantifying the relative positions of its vertices. Because they transform complex graph structures into numerical values, they provide an effective framework for comparing, analyzing, and interpreting different graph models. Over the years, distance-based topological indices have become an important area of study in graph theory due to their rich mathematical properties and their ability to characterize the structural complexity of graphs.

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The significance of these indices extends well beyond chemical graph theory and network analysis. In transportation and logistics, they assist in modeling road and railway systems, optimizing travel distances, and improving route planning. In communication and wireless networks, they are employed to evaluate network connectivity, transmission efficiency, and the robustness of communication infrastructures. In computer science, distance-based graph descriptors support the design and analysis of graph algorithms, database indexing, information retrieval, graph mining, and machine learning techniques involving graph-structured data. They are also useful in biological and ecological studies for investigating interaction networks, evolutionary relationships, and species connectivity. Furthermore, these indices have applications in social network analysis, where they help measure communication efficiency, identify influential vertices, and examine the overall structure of social interactions. Their usefulness has also expanded to fields such as image processing, geographic information systems, robotics, and infrastructure planning, where graph-based models play a fundamental role in solving real-world optimization and decision-making problems.

Owing to their broad applicability and strong mathematical foundation, distance-based topological indices continue to be an active area of research. The development of exact expressions for these indices on different graph families and graph products not only advances theoretical graph theory but also provides efficient mathematical tools for addressing practical problems across a wide range of scientific and engineering disciplines.

Definition 1.1. *If in a graph, every two distinct pair of vertices are joined by an edge, the graph is then said to be complete. In an m-partite graph, if each vertex in a partite set is adjacent to all the vertices in every other partite set then the graph is called as a complete m-partite graph.Figure 1 illustrates the complete graph* K_5 *where every pair of vertices is connected.*

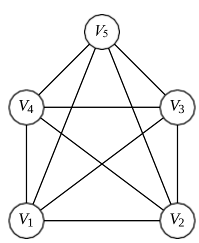

Figure 1: Complete Graph K_5

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Definition 1.2. *A* cycle *is a non-empty trail in which only the first and last vertices coincide. It is denoted by C_n. Figure 2 depicts the cycle graph C_4 , forming a closed loop of four vertices*

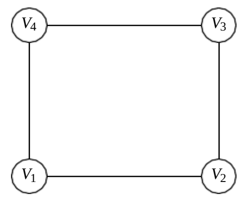

Figure 2: Cycle Graph C_4

Definition 1.3. *In [3,4,5], 1947 Harry Wiener introduced the Wiener index and is defined as,*

$$W(G) = \sum_{u,v \in V(G)} d(u,v)$$

Definition 1.4. *In [7], 1993 Milan Randi introduced Hyper-Wiener index and is defined as,*

$$WW(G) = \frac{1}{2} \sum_{u,v \in V(G)} [d(u,v) + d(u,v)^2]$$

Definition 1.5. *In [8,9] Plavi et. al., and Ivancine et. al., introduced Harary index independently and defined as,*

$$H(G) = \sum_{u,v \in V(G)} \frac{1}{d(u,v)}$$

Definition 1.6. *In [10,11] Ivancineet.al., introduced the Reciprocal Complementary Wiener index given by,*

$$RCW(G) = \sum_{u,v \in V(G)} \frac{1}{d + 1 - d(u,v)}$$

Definition 1.7. *In [13], 1975 Milan Randi introduced the Sum Connectivity index and is defined as,*

$$SCI(G) = \sum_{u,v \in V(G)} \frac{1}{d(u,v) + 1}$$

Definition 1.8. *In [14], 1981 Eduard Batschelet introduced the Batschelet index and is defined as,*

$$B(G) = \sum_{u,v \in V(G)} d(u,v)^2$$

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Definition 1.9. *In [1], 2000 Balaban introduced the Reverse Wiener index and is defined as,*

$$\wedge(G) = \frac{n(n-1)d}{2} - W(G)$$

where $n = |V(G)|$ and d is the diameter of G.

Definition 1.10. *In [12], 2010 the Reciprocal Reverse Wiener (RRW) index $R\wedge(G)$ of a connected graph G is defined as*

$$R \wedge (G) = \sum_{u,v \in V(G)} \frac{1}{d - d(u,v)}$$

where d is the diameter of a graph.

Definition 1.11. *In [16], 1998 Hosoya introduced the Wiener Polarity index $W_P(G)$, which counts the number of unordered vertex pairs separated by a distance of three in a connected graph G is defined as*

$$W_P(G) = |u, v \subseteq V(G) : d(u,v) = 3\}|$$

Definition 1.12. *In [17], 1969 Harary.F introduced the concept of Eccentricity index $\varepsilon(v)$, which counts the maximum diatance between v and any other vertex of a graph G is defined as*

$$\varepsilon(v) = \max_{u \in V(G)} d(u,v)$$

2. Indices of $K_m\{C_n\}$

In this paper, we compute some distance based topological indices of *cluster corona* product of $K_m\{C_n\}$.

Definition 2.1. *The cluster of $G\{H\}$ of two graphs G and H is defined as the graph obtained by taking one vertex of G and |V(G)| copies of rooted graph H and by identifying the root of the i^{th} vertex of G to every vertex in the i^{th} copy of H. See Figure 3*

Here $diam(G) = 5$ and the distance between any pair of vertices, from $1, 2, \cdots diam(G)$.
The number of 1 distance, pair of vertices, is $\frac{m(m-1)}{2} + m(2n+1)$.
The number of 2 distance, pair of vertices, is $m(m-1) + \frac{mn(n-3)}{2} + mn$.
The number of 3 distance, pair of vertices is $\frac{m(m-1)}{2} + mn(m-1)$.
The number of 4 distance, pair of vertices, is $mn(m-1)$.
The number of 5 distance, pair of vertices, is $\frac{m(m-1)n^2}{2}$. for $m \geq 2$ and $n \geq 3$. Using the above, we derive the following theorems. Then

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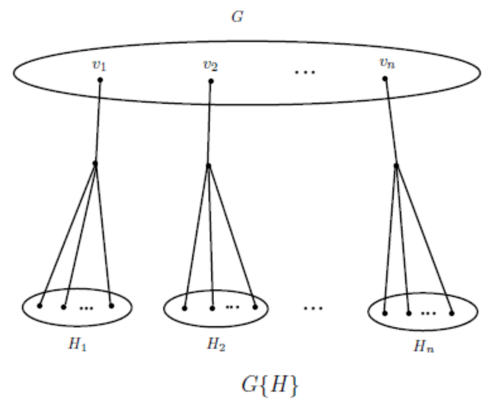

Figure 3: Cluster Product of two graphs G and H

Theorem 2.1. *Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$. Then*

(i) $W(K_m\{C_n\}) = \frac{m(m-1)}{2}\left[8 + 14n + 5n^2\right] + m\left[n^2 + n + 1\right]$

(ii) $WW(K_m\{C_n\}) = \frac{m(m-1)}{2}[13 + 32n + 15n^2] + \frac{m}{2}[n + 3n^2 + 2]$

Proof. Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$. Then $V(K_m\{C_n\}) = V_1 \cup V_2 \cup \cdots V_m$, where $V_i = \{v_{i,0}, v_{i,1}, \cdots v_{i,n-1}\}$, for $1 \leq i \leq m$. For $i, j = 1, 2, \ldots, m$ and $k, l = 0, 1, 2, \ldots, n-1$ the distance between any two vertices in $K_m\{C_n\}$ are given by

$$\begin{aligned}
d(v_i, v_j) &= 1, && i \neq j,\\
d(v_i, v_{i,1}) &= 1, &&\\
d(v_{i,k}, v_{i,l}) &= 1, && 1 \leq k \leq n-2,\\
d(v_i, v_{i,2}) &= 2, &&\\
d(v_{i,k}, v_{i,l}) &= 2, && 1 \leq k \leq n-3,\\
d(v_i, v_{j,1}) &= 2, && i \neq j,\\
d(v_{i,k}, v_{i,l}) &= 3, && 1 \leq k \leq n-4,\\
d(v_i, v_{j,2}) &= 3, && i \neq j,\\
d(v_{i,1}, v_{j,1}) &= 3, && i \neq j,\\
d(v_{i,k}, v_{i,l}) &= 4, && 1 \leq k \leq n-5,\\
d(v_{i,1}, v_{j,2}) &= 4, && i \neq j,\\
d(v_{i,k}, v_{i,l}) &= 5, && 1 \leq k \leq n-6,
\end{aligned}$$

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$$d(v_{i,2}, v_{j,2}) = 5, \qquad i \neq j.$$

(i) By the definition of Wiener Index

$$\begin{aligned}
W(K_m\{C_n\}) &= \sum_{u,v \in V(G)} d(u,v) \\
&= \left[\frac{m(m-1)}{2} + m(2n+1)\right](1) + \left[m(m-1) + \frac{mn(n-3)}{2} + mn\right](2) \\
&\quad + \left[\frac{m(m-1)}{2} + mn(m-1)\right](3) + [mn(m-1)](4) + \left[\frac{m(m-1)}{2}n^2\right](5) \\
&= \frac{m(m-1)}{2} + m(2n+1) + 2m(m-1) + mn(n-3) + 2mn + \frac{m(m-1)3}{2} \\
&\quad + 3mn(m-1) + 4mn(m-1) + \frac{5m(m-1)n^2}{2} \\
&= \frac{m(m-1)}{2}\left(1 + 4 + 3 + 6n + 8n + 5n^2\right) + m\left(2n + 1 + n^2 - 3n + 2n\right) \\
W(K_m\{C_n\}) &= \frac{m(m-1)}{2}\left(8 + 14n + 5n^2\right) + m\left(n^2 + n + 1\right)
\end{aligned}$$

(ii) By the definition of Hyper - Wiener Index

$$\begin{aligned}
WW(K_m\{C_n\}) &= \frac{1}{2}\sum_{u,v \in V(G)} \left[d(u,v) + d(u,v)^2\right] \\
&= \frac{1}{2}\left[\left(\frac{m(m-1)}{2} + m(2n+1)\right)(1+1^2) + \left(m(m-1) + \frac{mn(n-3)}{2} + mn\right)(2+2^2)\right. \\
&\quad + \left(\frac{m(m-1)}{2} + mn(m-1)\right)(3+3^2) + [mn(m-1)](4+4^2) \\
&\quad \left. + \left(\frac{m(m-1)}{2}n^2\right)(5+5^2)\right] \\
&= \frac{1}{2}\{m(m-1) + 2m(2n+1) + 6m(m-1) + 3mn(n-3) + 6mn + 6m(m-1) \\
&\quad + 12mn(m-1) + 20mn(m-1) + 15m(m-1)n^2\} \\
&= \frac{m(m-1)}{2}[1 + 6 + 6 + 12n + 20n + 15n^2] + \frac{1}{2}[4mn + 2m + 3mn^2 - 9mn + 6mn] \\
WW(K_m\{C_n\}) &= \frac{m(m-1)}{2}[13 + 32n + 15n^2] + \frac{m}{2}[n + 3n^2 + 2]
\end{aligned}$$

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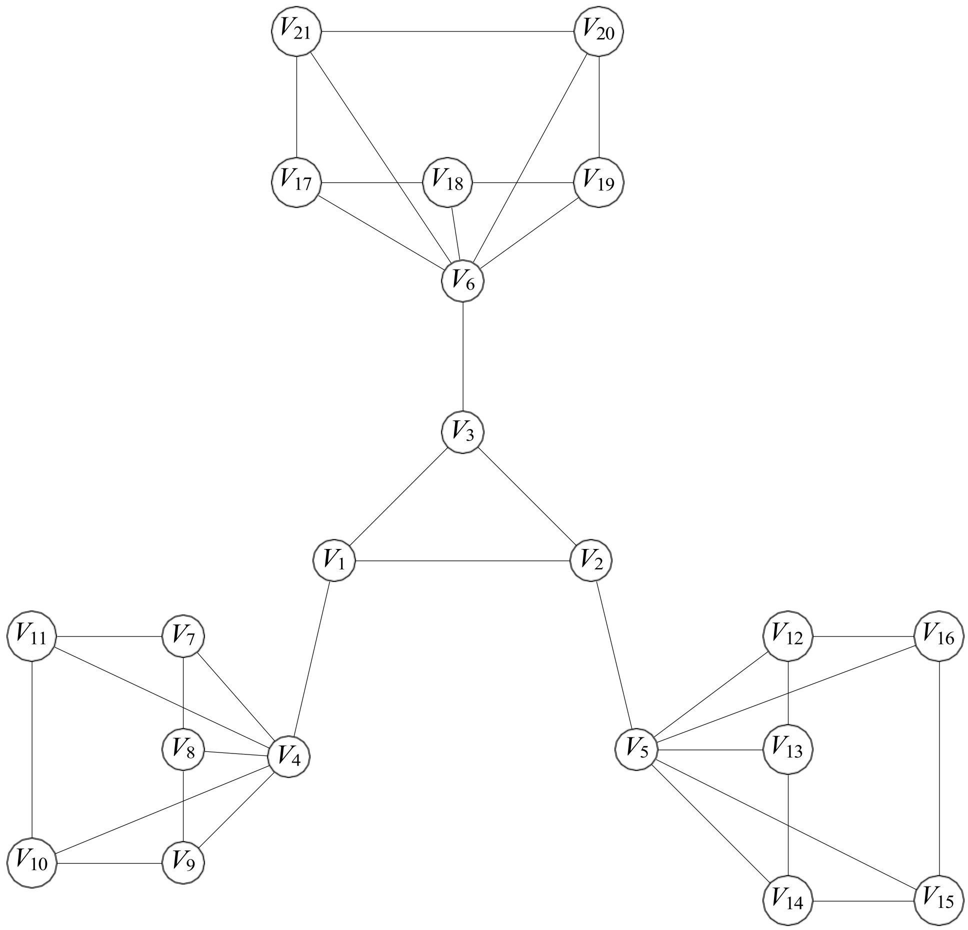

Figure 4: $K_3 \circ \{C_5\}$

Theorem 2.2. *Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$. Then*

(i) $H(K_m\{C_n\}) = \frac{m(m-1)}{2}\left[\frac{6n^2+35n+70}{30}\right] + m\left[\frac{n^2+7n+4}{4}\right]$

(ii) $RCW(K_m\{C_n\}) = \frac{m(m-1)}{2}\left[\frac{30n^2+50n+31}{30}\right] + m\left[\frac{5n^2+11n+8}{40}\right]$

Proof. Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$.

(i) By the definition of Harary Index

$$H(K_m\{C_n\}) = \sum_{u,v\in V(G)} \frac{1}{d(u,v)}$$

$$= \left[\frac{m(m-1)}{2} + m(2n+1)\right]\frac{1}{1} + \left[m(m-1) + \frac{mn(n-3)}{2} + mn\right]\frac{1}{2}$$

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$$+\left[\frac{m(m-1)}{2}+mn(m-1)\right]\frac{1}{3}+[mn(m-1)]\frac{1}{4}+\left[\frac{m(m-1)}{2}n^2\right]\frac{1}{5}$$

$$=\frac{m(m-1)}{2}+m(2n+1)+\frac{m(m-1)}{2}+\frac{mn(n-3)}{4}+\frac{mn}{2}+\frac{m(m-1)}{6}+\frac{mn(m-1)}{3}+\frac{mn(m-1)}{4}+\frac{m(m-1)n^2}{10}$$

$$=\frac{m(m-1)}{2}\left[1+1+\frac{1}{3}+\frac{2n}{3}+\frac{n}{2}+\frac{n^2}{5}\right]+m\left[2n+1+\frac{n^2}{4}-\frac{3n}{4}+\frac{n}{2}\right]$$

$$=\frac{m(m-1)}{2}\left[\frac{n^2}{5}+\frac{7n}{6}+\frac{7}{3}\right]+m\left[\frac{n^2}{4}+\frac{7n}{4}+1\right]$$

$$H(K_m\{C_n\})=\frac{m(m-1)}{2}\left[\frac{6n^2+35n+70}{30}\right]+m\left[\frac{n^2+7n+4}{4}\right]$$

(ii) By the definition of Reciprocal Complementary Wiener Index

$$RCW(K_m\{C_n\})=\sum_{u,v\in V(G)}\frac{1}{d+1-d(u,v)}$$

$$=\left[\frac{m(m-1)}{2}+m(2n+1)\right]\frac{1}{5}+\left[m(m-1)+\frac{mn(n-3)}{2}+mn\right]\frac{1}{4}$$

$$+\left[\frac{m(m-1)}{2}+mn(m-1)\right]\frac{1}{3}+[mn(m-1)]\frac{1}{2}+\left[\frac{m(m-1)}{2}n^2\right]\frac{1}{1}$$

$$=\frac{m(m-1)}{10}+\frac{m(2n+1)}{5}+\frac{m(m-1)}{4}+\frac{mn(n-3)}{8}+\frac{mn}{4}+\frac{m(m-1)}{6}+\frac{mn(m-1)}{3}+\frac{mn(m-1)}{2}+\frac{m(m-1)n^2}{2}$$

$$=\frac{m(m-1)}{2}\left[\frac{1}{5}+\frac{1}{2}+\frac{1}{3}+\frac{2n}{3}+n+n^2\right]+m\left[\frac{2n+1}{5}+\frac{n^2-3n}{8}+\frac{n}{4}\right]$$

$$=\frac{m(m-1)}{2}\left[n^2+\frac{5n}{3}+\frac{6+15+10}{30}\right]+m\left[\frac{5n^2+16n-15n+8+10n}{40}\right]$$

$$RCW(K_m\{C_n\})=\frac{m(m-1)}{2}\left[\frac{30n^2+50n+31}{30}\right]+m\left[\frac{5n^2+11n+8}{40}\right]$$

Theorem 2.3. *Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$. Then*

(i) $SCI(K_m\{C_n\})=\frac{m(m-1)}{2}\left[\frac{10n^2+54n+85}{60}\right]+m\left[\frac{n^2+5n+3}{6}\right]$

(ii) $B(K_m\{C_n\})=\frac{m(m-1)}{2}[25n^2+50n+18]+m[2n^2+1]$

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Proof. Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$.

(i) By the definition of Sum Connectivity Index

$$
\begin{aligned}
SCI(K_m\{C_n\}) &= \sum_{u,v \in V(G)} \frac{1}{d(u,v)+1} \\
&= \left(\frac{m(m-1)}{2} + m(2n+1)\right)\left(\frac{1}{1+1}\right) + \left(m(m-1) + \frac{mn(n-3)}{2} + mn\right)\left(\frac{1}{2+1}\right) \\
&\quad + \left(\frac{m(m-1)}{2} + mn(m-1)\right)\left(\frac{1}{3+1}\right) + [mn(m-1)]\left(\frac{1}{4+1}\right) \\
&\quad + \left(\frac{m(m-1)n^2}{2}\right)\left(\frac{1}{5+1}\right) \\
&= \frac{m(m-1)}{4} + \frac{m(2n+1)}{2} + \frac{m(m-1)}{3} + \frac{mn(n-3)}{6} + \frac{mn}{3} + \frac{m(m-1)}{8} \\
&\quad + \frac{mn(m-1)}{4} + \frac{mn(m-1)}{5} + \frac{m(m-1)n^2}{12} \\
&= \frac{m(m-1)}{2}\left(\frac{1}{2} + \frac{2}{3} + \frac{1}{4} + \frac{n}{2} + \frac{2n}{5} + \frac{n^2}{6}\right) + m\left(\frac{2n+1}{2} + \frac{n}{3} + \frac{n^2-3n}{6}\right) \\
&= \frac{m(m-1)}{2}\left(\frac{30+40+15+30n+24n+10n^2}{60}\right) + m\left(\frac{n^2-3n+3+2n+6n}{6}\right) \\
SCI(K_m\{C_n\}) &= \frac{m(m-1)}{2}\left(\frac{10n^2+54n+85}{60}\right) + m\left(\frac{n^2+5n+3}{6}\right)
\end{aligned}
$$

(ii) By the definition of Batschelet Index

$$
\begin{aligned}
B(K_m\{C_n\}) &= \sum_{u,v \in V(G)} d(u,v)^2 \\
&= \left(\frac{m(m-1)}{2} + m(2n+1)\right)(1^2) + \left(m(m-1) + \frac{mn(n-3)}{2} + mn\right)(2^2) \\
&\quad + \left(\frac{m(m-1)}{2} + mn(m-1)\right)(3^2) + [mn(m-1)](4^2) + \left(\frac{m(m-1)}{2}n^2\right)(5^2) \\
&= \frac{m(m-1)}{2} + m(2n+1) + 4m(m-1) + 2mn(n-3) + 4mn + \frac{9m(m-1)}{2} \\
&\quad + 9mn(m-1) + 16mn(m-1) + \frac{25m(m-1)n^2}{2} \\
&= \frac{m(m-1)}{2}\left(1+8+18n+9+32n+25n^2\right) + m[2n^2+1+2n-6n+4n] \\
B(K_m\{C_n\}) &= \frac{m(m-1)}{2}[25n^2+50n+18] + m[2n^2+1]
\end{aligned}
$$

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Theorem 2.4. *Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$. Then*

(i) $\wedge(K_m\{C_n\}) = \frac{m}{2}\left\{5(n+1)[m(n+1)-1] - [(m-1)(5n^2+14n+8) - 2(n^2+n+1)]\right\}$

(ii) $R \wedge (K_m\{C_n\}) = \frac{m(m-1)}{2}\left(\frac{36n+17}{12}\right) + m\left[\frac{2n^2+4n+3}{12}\right]$

Proof. Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$.

(i) By the definition of Reverse Wiener Index

Here, $n = m + mn$

$$\begin{aligned}
\wedge(K_m\{C_n\}) &= \frac{n(n-1)d}{2} - W(G) \\
&= \frac{(m+mn)((m+mn)-1)5}{2} - \frac{m(m-1)}{2}(5n^2+14n+8) - m(n^2+n+1) \\
&= 5\left(\frac{m^2+m^2n-m+m^2n+m^2n^2-mn}{2}\right) - \frac{m(m-1)}{2}(5n^2+14n+8) - m(n^2+n+1) \\
&= 5\left(\frac{m^2(1+2n+n^2)-m(n+1)}{2}\right) - \frac{m(m-1)}{2}(5n^2+14n+8) - m(n^2+n+1) \\
&= \frac{m}{2}\left\{5[m(n+1)^2-(n+1)] - [(m-1)(5n^2+14n+8) - 2(n^2+n+1)]\right\} \\
\wedge(K_m\{C_n\}) &= \frac{m}{2}\left\{5(n+1)[m(n+1)-1] - [(m-1)(5n^2+14n+8) - 2(n^2+n+1)]\right\}
\end{aligned}$$

(ii) By the definition of Reciprocal Reverse Wiener Index

$$\begin{aligned}
R \wedge (K_m\{C_n\}) &= \sum_{u,v \in V(G)} \frac{1}{d - d(u,v)} \\
&= \left[\frac{m(m-1)}{2} + m(2n+1)\right]\left(\frac{1}{5-1}\right) + \left[m(m-1) + \frac{mn(n-3)}{2} + mn\right]\left(\frac{1}{5-2}\right) \\
&\quad + \left[\frac{m(m-1)}{2} + mn(m-1)\right]\left(\frac{1}{5-3}\right) + [mn(m-1)]\left(\frac{1}{5-4}\right) \\
&\quad + \left[\frac{m(m-1)}{2}n^2\right]\left(\frac{1}{5-5}\right) \\
&= \frac{m(m-1)}{8} + \frac{m(2n+1)}{4} + \frac{m(m-1)}{3} + \frac{mn(n-3)}{6} + \frac{mn}{3} + \frac{m(m-1)}{4} + \frac{mn(m-1)}{2} \\
&\quad + mn(m-1) \\
&= \frac{m(m-1)}{2}\left(\frac{1}{4} + \frac{2}{3} + \frac{1}{2} + n + 2n\right) + m\left(\frac{2n+1}{4} + \frac{n^2-3n}{6} + \frac{n}{3}\right)
\end{aligned}$$

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$$= \frac{m(m-1)}{2}\left(\frac{3+8+6+12n+24n}{12}\right) + m\left(\frac{2n^2-6n+3+4n+6n}{12}\right)$$

$$R_{\wedge}(K_m\{C_n\}) = \frac{m(m-1)}{2}\left(\frac{36n+17}{12}\right) + m\left(\frac{2n^2+4n+3}{12}\right)$$

Theorem 2.5. *Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$. Then*

(i) $W_P(K_m\{C_n\}) = m^2\left(\frac{2n+1}{2}\right) - m\left(\frac{2n+1}{2}\right)$

(ii) $\varepsilon(K_m\{C_n\}) = \frac{1}{2}\left(m^2n^2 - mn^2\right)$

Proof. Let K_m and C_n be complete and the cycle graphs with $m \geq 2$ and $n \geq 3$.

(i) By the definition of Wiener Polarity Index

$$\begin{aligned} W_P(G) &= \{\{u,v\} \subseteq V(G)/d(u,v) = 3\} \\ &= \frac{m(m-1)}{2} + mn(m-1) \\ &= \frac{m^2-m}{2} + m^2n - mn \\ &= m^2\left(n+\frac{1}{2}\right) - m\left(n+\frac{1}{2}\right) \\ W_P(G) &= m^2\left(\frac{2n+1}{2}\right) - m\left(\frac{2n+1}{2}\right) \end{aligned}$$

(ii) By the definition of Eccentricity Index

$$\begin{aligned} \varepsilon(v) &= max\{d(i,j)\} \text{ for all Vertices j} \\ &= \frac{m(m-1)n^2}{2} \\ &= \frac{m^2n^2 - mn^2}{2} \\ \varepsilon(v) &= \frac{1}{2}\left(m^2n^2 - mn^2\right) \end{aligned}$$

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Conclusion

This research established closed-form expressions for several distance-based topological indices associated with the cluster product of complete graphs and cycle graphs. The obtained results were derived by carefully analyzing the distance relationships among the vertices of the constructed graph. These mathematical expressions enrich the existing body of knowledge on graph products and provide a useful foundation for the quantitative analysis of network structures.

There remain numerous opportunities to extend this work. Future investigations may include the evaluation of other distance-oriented graph invariants for the same graph product, as well as the study of related graph operations involving different classes of graphs. Similar methodologies can also be applied to cluster products formed from paths, complete bipartite graphs, stars, wheels, and other graph families. Moreover, comparing the behavior of various topological descriptors across different graph products may lead to a deeper understanding of their structural characteristics and support further applications in chemical graph theory, communication networks, and complex systems.

References

[1] D. B. West, *Introduction to Graph Theory* (Upper Saddle River, NJ: Prentice-Hall, 1996).

[2] P. Et al, *"Distance Based Topological Indices and Regular Graphs*", Turkish Journal of Computer and Mathematics Education 12, no. 3 (2021): 5191–5196,

[3] K. Pattabiraman, *"Degree and Distance Based Topological Indices of Graphs,"* Electronic Notes in Discrete Mathematics 63, no. 5 (2017): 145–159,

[4] S. Stevanovic and D. Stevanovic, *"On Distance-Based To pological Indices Used in Architectural Research,"* MATCH Commun. Math. Comput. Chem. 79, no. 7 (2018): 659–683.

[5] A. A. Dobrynin, R. Entringer, and I. Gutman, *"Wiener Index of Trees: Teory and Applications,"* Acta Applicandae Mathematica 66, no. 3 (2001): 211–249,

[6] D. Bonchev, *"Te Wiener Number-Some Applications and NewDevelopments,"* in Topology in Chemistry (Sawston, UK: Woodhead Publishing, 2002), 58–88.

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/3/004

[7] L. Chen,A.Mehboob,H.Ahmad,W.Nazeer,M.Hussain,and M. R. Farahani, *"Hosoya and Harary Polynomials of and,"* Discrete Dynamics in Nature and Society 20, no. 19 (2019): 23–36.

[8] G. Zhou, H. Li, R. Song, Q. Wang, J. Xu, and B. Song, *"Orthorectifcation of Fisheye Image Under Equidistant Projection Model,"* Remote Sensing 14, no. 17 (2022): 4175,

[9] N. Tomas, L. Mathew, S. Sriram, A. K. Nagar, and K. G. Subramanian, "Certain Distance-Based Topological Indices of Parikh Word Representable Graphs," Journal of Mathematics 2021, no. 1 (2021): 1–7,

[10] M. A. Malik, M. Imran, and M. Adeel, *"On Distance-Based Topological Indices and Co-indices of Fractal-Type Molecular Graphs and Teir Respective Graph Entropies,"* PLoS One 18, no. 11 (2023): e0290047–e0290059,

[11] R. Balakrishnan, K. Ranganathan, *A Textbook of Graph Theory, Springer Science,* New York,(2012).

[12] J.A.Bondy,U.S.R.Murty,*Graph Theory with Applications, Macmillan,* NewYork,(1976).