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A HYBRID CUBIC BIPOLAR PYTHAGOREAN FUZZY MCDM MODEL INTEGRATING ENTROPY, VIKOR AND COPRAS METHODS

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Abstract: In this work, a hybrid multicriteria decision making (MCDM) model utilizing the Cubic Bipolar Pythagorean Fuzzy Sets (CBPFSs) in the R-order has been presented. Interior, Exterior, Frontier, and Basis have been introduced to construct a topological structure. Entropy is used to calculate the objective criteria weights, then VIKOR and COPRAS methods are adopted for ranking of alternatives. The proposed methodology is illustrated via a real-world problem related to selection of the best alternative for cancer treatment.

Keywords: CBPFS, R-order, Entropy, VIKOR, COPRAS, MCDM, Cancer Treatment.

I. INTRODUCTION

Multi Criteria Decision Making (MCDM) serves as an effective tool for choosing the most appropriate alternative in case of having several criteria that contradict each other. Since, in practice, the information regarding decision-making is uncertain, imprecise and cannot be measured precisely in numeric values, it becomes necessary to use fuzzy set theory [1] and its generalizations for representation of the uncertain information in the decision-making problem.

The Pythagorean fuzzy sets (PFSs) [3] have more flexibility since, in contrast to intuitionistic fuzzy sets, they allow the sum of the squared membership and non-membership degrees to be at most one. Adding bipolarity and intervals as additional types of information results in the formation of the Cubic Bipolar Pythagorean Fuzzy Sets (CBPFSs). Thus, they become able to reflect not only positive but also negative uncertain information.

In this paper, CBPFSs will be considered according to the R-order. Using this order, the interior, exterior, frontier and basis will be defined, thus, creating the corresponding topological structure. Thus, these notions can serve as a mathematical background for investigation of the structural properties of the CBPFSs.

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For the MCDM process, the **entropy method** is used to determine the objective weights of the criteria. The **VIKOR [5]** method is then employed to obtain a compromise ranking, while **COPRAS [13]** is used to evaluate the relative significance of alternatives considering beneficial and non-beneficial criteria. Thus, the proposed framework combines CBPFS, entropy weighting, VIKOR and COPRAS into a unified decision-making methodology.

Finally, the proposed method is applied to a **cancer-treatment selection problem**, where several treatment alternatives are evaluated under multiple clinical criteria. The application demonstrates the usefulness of the proposed framework in handling uncertain and conflicting information in healthcare decision-making.

II LITERATURE REVIEW

Zadeh [1] presented the theory of fuzzy sets to deal with imprecise data that then gave rise to many extensions for MCDM problems. Yager [2] presented the concept of Pythagorean fuzzy sets, offering more flexibility in depicting membership and non-membership functions. Cubic and bipolar fuzzy sets later contributed to the inclusion of interval-valued, exact, and both positive and negative information at once.

There are numerous MCDM[8] techniques in different fuzzy domains. Entropy technique is often employed to determine the criterion weights through the information available in the decision matrix. The VIKOR approach, presented by Opricovic and Tzeng [3], offers a compromise solution to conflicting criteria, while COPRAS ranking technique considers both beneficial and non-beneficial criteria to rank alternatives.

Entropy-based fuzzy VIKOR and COPRAS approaches have been successfully applied to various selection and healthcare problems. However, comparatively limited attention has been given to integrating **Cubic Bipolar Pythagorean Fuzzy Sets with R-order topological concepts and hybrid entropy–VIKOR–COPRAS decision-making**.

Therefore, this study develops a hybrid CBPFS-based MCDM framework incorporating the R-order concepts of **interior, exterior, frontier and basis**, objective entropy weighting, and VIKOR and COPRAS ranking techniques. Its applicability is demonstrated through a cancer-treatment selection problem.

III PRELIMINARIES

Definition 2.1

Let $\mathcal{M}$ be a universe of discourse. A **cubic bipolar Pythagorean fuzzy set (CBPFS)** C_{BP} on universe set $\mathcal{M}$ is defined as follows

$$C_{BP} = \{(x, C(x), \sigma(x)) \mid x \in \mathcal{M}\}$$

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where $C(x) = \{(x, [(\mu^{+l}_{C_{BP}}, \mu^{+u}_{C_{BP}}), (\vartheta^{+l}_{C_{BP}}, \vartheta^{+u}_{C_{BP}})], [(\mu^{-l}_{C_{BP}}, \mu^{-u}_{C_{BP}}), (\vartheta^{-l}_{C_{BP}}, \vartheta^{-u}_{C_{BP}})]) \mid x \in \mathcal{M}\}$ represents the IVBPFS defined on $\mathcal{M}$ while $\sigma(x) = \{(x, [\mu^{+}_{C_{BP}}, \vartheta^{+}_{C_{BP}}], [\mu^{-}_{C_{BP}}, \vartheta^{-}_{C_{BP}}]) \mid x \in \mathcal{M}\}$ represents the BPFS defined on $\mathcal{M}$ in which the mappings $\mu^{+}_{C_{BP}} : \mathcal{M} \to [0, 1]$ denote the degree of positive membership, $\vartheta^{+}_{C_{BP}} : \mathcal{M} \to [0, 1]$ denote the degree of positive non-membership, $\mu^{-}_{C_{BP}} : \mathcal{M} \to [-1, 0]$ denote the degree of negative membership and $\vartheta^{-}_{C_{BP}} : \mathcal{M} \to [-1, 0]$ denote the degree of negative non-membership of each element $x \in \mathcal{M}$ to the set P_B such that $(\mu^{+l}_{C_{BP}}, \mu^{+u}_{C_{BP}}) \subset [0,1]$, $(\vartheta^{+l}_{C_{BP}}, \vartheta^{+u}_{C_{BP}}) \subset [0,1]$, $(\mu^{-l}_{C_{BP}}, \mu^{-u}_{C_{BP}}) \subset [-1, 0]$, $(\vartheta^{-l}_{C_{BP}}, \vartheta^{-u}_{C_{BP}}) \subset [-1, 0]$ and $\mu^{+l}_{C_{BP}} \leq \mu^{+u}_{C_{BP}}$, $\mu^{-l}_{C_{BP}} \leq \mu^{-u}_{C_{BP}}$, $\vartheta^{+l}_{C_{BP}} \leq \vartheta^{+u}_{C_{BP}}$ $\vartheta^{-l}_{C_{BP}} \leq \vartheta^{-u}_{C_{BP}}$.

Also, $0 \leq (\mu^{+u}_{C_{BP}})^2 + (\vartheta^{+u}_{C_{BP}})^2 \leq 1$, $0 \leq (\mu^{-u}_{C_{BP}})^2 + (\vartheta^{-u}_{C_{BP}})^2 \leq 1$.

$0 \leq (\mu^{+l}_{C_{BP}})^2 + (\vartheta^{+l}_{C_{BP}})^2 \leq 1$, $0 \leq (\mu^{-l}_{C_{BP}})^2 + (\vartheta^{-l}_{C_{BP}})^2 \leq 1$

and $0 \leq (\mu^{+}_{C_{BP}})^2 + (\vartheta^{+}_{C_{BP}})^2 \leq 1$, $0 \leq (\mu^{-}_{C_{BP}})^2 + (\vartheta^{-}_{C_{BP}})^2 \leq 1$.

We denote this pair $C_{BP} = (C, \sigma)$ where

$C = ([(\mu^{+l}_{C_{BP}}, \mu^{+u}_{C_{BP}}), (\vartheta^{+l}_{C_{BP}}, \vartheta^{+u}_{C_{BP}})], [(\mu^{-l}_{C_{BP}}, \mu^{-u}_{C_{BP}}), (\vartheta^{-l}_{C_{BP}}, \vartheta^{-u}_{C_{BP}})])$ and

$\sigma = [\mu^{+}_{C_{BP}}, \vartheta^{+}_{C_{BP}}, \mu^{-}_{C_{BP}}, \vartheta^{-}_{C_{BP}}]$ is known as cubic bipolar Pythagorean fuzzy number (CBPFN).

Let $\pi_{C_{BP}}(x) = ([\pi^{+l}_{C_{BP}}(x), \pi^{+u}_{C_{BP}}(x)], [\pi^{-l}_{C_{BP}}(x), \pi^{-u}_{C_{BP}}(x)]; [\pi^{l}_{C_{BP}}(x), \pi^{u}_{C_{BP}}(x)])$ for all x in $\mathcal{M}$, then π_{P_C} is called the CBPF index of x to $\mathcal{M}$, where

$\pi^{+l}_{C_{BP}}(x) = \sqrt{1 - (\mu^{+l}_{C_{BP}}(x))^2 - (\vartheta^{+l}_{C_{BP}}(x))^2}$, $\pi^{+u}_{C_{BP}}(x) = \sqrt{1 - (\mu^{+u}_{C_{BP}}(x))^2 - (\vartheta^{+u}_{C_{BP}}(x))^2}$, $\pi^{-l}_{C_{BP}}(x) = \sqrt{1 - (\mu^{-l}_{C_{BP}}(x))^2 - (\vartheta^{-l}_{C_{BP}}(x))^2}$, $\pi^{+u}_{C_{BP}}(x) = \sqrt{1 - (\mu^{-u}_{C_{BP}}(x))^2 - (\vartheta^{-u}_{C_{BP}}(x))^2}$ and $\pi^{l}_{C_{BP}}(x) = \sqrt{1 - (\mu^{l}_{C_{BP}}(x))^2 - (\vartheta^{l}_{C_{BP}}(x))^2}$, $\pi^{u}_{C_{BP}}(x) = \sqrt{1 - (\mu^{u}_{C_{BP}}(x))^2 - (\vartheta^{u}_{C_{BP}}(x))^2}$.

Definition 2.2

Let $C_{BP_1} = (C_1, \sigma_1)$ and $C_{BP_2} = (C_2, \sigma_2)$ be two CBPFSs defined over the universal set $\mathcal{M}$ in which

$C_1 = ([(\mu^{+l}_{C_{BP_1}}, \mu^{+u}_{C_{BP_1}}), (\vartheta^{+l}_{C_{BP_1}}, \vartheta^{+u}_{C_{BP_1}})], [(\mu^{-l}_{C_{BP_1}}, \mu^{-u}_{C_{BP_1}}), (\vartheta^{-l}_{C_{BP_1}}, \vartheta^{-u}_{C_{BP_1}})])$,

$\sigma_1 = [\mu^{+}_{C_{BP_1}}, \vartheta^{+}_{C_{BP_1}}, \mu^{-}_{C_{BP_1}}, \vartheta^{-}_{C_{BP_1}}]$

$C_2 = ([(\mu^{+l}_{C_{BP_2}}, \mu^{+u}_{C_{BP_2}}), (\vartheta^{+l}_{C_{BP_2}}, \vartheta^{+u}_{C_{BP_2}})], [(\mu^{-l}_{C_{BP_2}}, \mu^{-u}_{C_{BP_2}}), (\vartheta^{-l}_{C_{BP_2}}, \vartheta^{-u}_{C_{BP_2}})])$ and

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$\sigma_2 = \left[\mu^{+}_{C_{BP_2}}, \vartheta^{+}_{C_{BP_2}}, \mu^{-}_{C_{BP_2}}, \vartheta^{-}_{C_{BP_2}}\right]$.

Then we define:

Equality $C_{BP_1} = C_{BP_2}$ if $\left(\mu^{+l}_{C_{BP_1}}, \mu^{+u}_{C_{BP_1}}\right) = \left(\mu^{+l}_{C_{BP_2}}, \mu^{+u}_{C_{BP_2}}\right)$, $\left(\vartheta^{+l}_{C_{BP_1}}, \vartheta^{+u}_{C_{BP_1}}\right) = \left(\vartheta^{+l}_{C_{BP_2}}, \vartheta^{+u}_{C_{BP_2}}\right)$, $\left(\mu^{-l}_{C_{BP_1}}, \mu^{-u}_{C_{BP_1}}\right) = \left(\mu^{-l}_{C_{BP_2}}, \mu^{-u}_{C_{BP_2}}\right)$, $\left(\vartheta^{-l}_{C_{BP_1}}, \vartheta^{-u}_{C_{BP_1}}\right) = \left(\vartheta^{-l}_{C_{BP_2}}, \vartheta^{-u}_{C_{BP_2}}\right)$, $\mu^{+}_{C_{BP_1}} = \mu^{+}_{C_{BP_2}}$, $\vartheta^{+}_{C_{BP_1}} = \vartheta^{+}_{C_{BP_2}}$, $\mu^{-}_{C_{BP_1}} = \mu^{-}_{C_{BP_2}}$ and $\vartheta^{-}_{C_{BP_1}} = \vartheta^{-}_{C_{BP_2}}$.

$\mathbb{R}$**-order** $C_{BP_1} \subseteq_{\mathbb{R}} C_{BP_2}$ if $\left(\mu^{+l}_{C_{BP_1}}, \mu^{+u}_{C_{BP_1}}\right) \subseteq \left(\mu^{+l}_{C_{BP_2}}, \mu^{+u}_{C_{BP_2}}\right)$, $\left(\vartheta^{+l}_{C_{BP_1}}, \vartheta^{+u}_{C_{BP_1}}\right) \supseteq \left(\vartheta^{+l}_{C_{BP_2}}, \vartheta^{+u}_{C_{BP_2}}\right)$, $\left(\mu^{-l}_{C_{BP_1}}, \mu^{-u}_{C_{BP_1}}\right) \subseteq \left(\mu^{-l}_{C_{BP_2}}, \mu^{-u}_{C_{BP_2}}\right)$, $\left(\vartheta^{-l}_{C_{BP_1}}, \vartheta^{-u}_{C_{BP_1}}\right) \supseteq \left(\vartheta^{-l}_{C_{BP_2}}, \vartheta^{-u}_{C_{BP_2}}\right)$, $\mu^{+}_{C_{BP_1}} \geq \mu^{+}_{C_{BP_2}}$, $\vartheta^{+}_{C_{BP_1}} \leq \vartheta^{+}_{C_{BP_2}}$, $\mu^{-}_{C_{BP_1}} \geq \mu^{-}_{C_{BP_2}}$ and $\vartheta^{-}_{C_{BP_1}} \leq \vartheta^{-}_{C_{BP_2}}$.

Definition 2.3

Let $C_{BP_i} = (C_i, \sigma_i)$ where $i \in \Lambda$, be a family of CBPFSs defined over the set $\mathscr{M}$. Then we define:

(i) $\mathbb{R}$**-union:**

$$\bigcup_{\mathbb{R}} C_{BP_i}$$

$$= \left\{\left[\left(\sup_{i\in\Lambda} \mu^{+l}_{C_{BP_i}}, \sup_{i\in\Lambda} \mu^{+u}_{C_{BP_i}}\right), \left(\inf_{i\in\Lambda} \vartheta^{+l}_{C_{BP_i}}, \inf_{i\in\Lambda} \vartheta^{+u}_{C_{BP_i}}\right), \left(\inf_{i\in\Lambda} \mu^{-l}_{C_{BP_i}}, \inf_{i\in\Lambda} \mu^{-u}_{C_{BP_i}}\right), \left(\sup_{i\in\Lambda} \vartheta^{-l}_{C_{BP_i}}, \sup_{i\in\Lambda} \vartheta^{-u}_{C_{BP_i}}\right)\right],\right.$$
$$\left.\left[\inf_{i\in\Lambda} \mu^{+}_{C_{BP_i}}, \sup_{i\in\Lambda} \vartheta^{+}_{C_{BP_i}}, \sup_{i\in\Lambda} \mu^{-}_{C_{BP_i}}, \inf_{i\in\Lambda} \vartheta^{-}_{C_{BP_i}}\right]\right\}$$

(ii) $\mathbb{R}$**-intersection:**

$$\bigcap_{\mathbb{R}} C_{BP_i}$$

$$= \left\{\left[\left(\inf_{i\in\Lambda} \mu^{+l}_{C_{BP_i}}, \inf_{i\in\Lambda} \mu^{+u}_{C_{BP_i}}\right), \left(\sup_{i\in\Lambda} \vartheta^{+l}_{C_{BP_i}}, \sup_{i\in\Lambda} \vartheta^{+u}_{C_{BP_i}}\right), \left(\sup_{i\in\Lambda} \mu^{-l}_{C_{BP_i}}, \sup_{i\in\Lambda} \mu^{-u}_{C_{BP_i}}\right), \left(\inf_{i\in\Lambda} \vartheta^{-l}_{C_{BP_i}}, \inf_{i\in\Lambda} \vartheta^{-u}_{C_{BP_i}}\right)\right],\right.$$
$$\left.\left[\sup_{i\in\Lambda} \mu^{+}_{C_{BP_i}}, \inf_{i\in\Lambda} \vartheta^{+}_{C_{BP_i}}, \inf_{i\in\Lambda} \mu^{-}_{C_{BP_i}}, \sup_{i\in\Lambda} \vartheta^{-}_{C_{BP_i}}\right]\right\}$$

Definition 2.4

Let $C_{BP} = (C, \sigma)$ in which $C = \left(\left[\left(\mu^{+l}_{C_{BP}}, \mu^{+u}_{C_{BP}}\right), \left(\vartheta^{+l}_{C_{BP}}, \vartheta^{+u}_{C_{BP}}\right)\right], \left[\left(\mu^{-l}_{C_{BP}}, \mu^{-u}_{C_{BP}}\right), \left(\vartheta^{-l}_{C_{BP}}, \vartheta^{-u}_{C_{BP}}\right)\right]\right)$ and $\sigma = \left[\mu^{+}_{C_{BP}}, \vartheta^{+}_{C_{BP}}, \mu^{-}_{C_{BP}}, \vartheta^{-}_{C_{BP}}\right]$ be a CBPFS defined over the universe $\mathscr{M}$. Then, the **complement** of C_{BP} can be defined as $(C_{BP})^C = (C^C, \sigma^C)$ where $C^C = \left(\left[\left(1 - \mu^{+u}_{C_{BP}}, 1 - \mu^{+l}_{C_{BP}}\right), \left(1 - \vartheta^{+u}_{C_{BP}}, 1 - \vartheta^{+l}_{C_{BP}}\right)\right], \left[\left(-1 - \mu^{-u}_{C_{BP}}, -1 - \mu^{-l}_{C_{BP}}\right), \left(-1 - \vartheta^{-u}_{C_{BP}}, -1 - \vartheta^{-l}_{C_{BP}}\right)\right]\right)$ and $\sigma^C = \left[1 - \mu^{+}_{C_{BP}}, 1 - \vartheta^{+}_{C_{BP}}, -1 - \mu^{-}_{C_{BP}}, -1 - \vartheta^{-}_{C_{BP}}\right]$.

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Theorem 2.5

Let $C_{BP_1} = (C_1, \sigma_1)$ and $C_{BP_2} = (C_2, \sigma_2)$ be two CBPFSs defined over the universal set $\mathscr{M}$. Then the following properties are hold under $\mathbb{R}$-order:

(i) $C_{BP_1} \cup_{\mathbb{R}} C_{BP_2} = C_{BP_2} \cup_{\mathbb{R}} C_{BP_1}$

(ii) $C_{BP_1} \cap_{\mathbb{R}} C_{BP_2} = C_{BP_2} \cap_{\mathbb{R}} C_{BP_1}$

(iii) $\left(C_{BP_1} \cup_{\mathbb{R}} C_{BP_2}\right)^C = \left(C_{BP_1}\right)^C \cap_{\mathbb{R}} \left(C_{BP_2}\right)^C$

(iv) $\left(C_{BP_1} \cap_{\mathbb{R}} C_{BP_2}\right)^C = \left(C_{BP_1}\right)^C \cup_{\mathbb{R}} \left(C_{BP_2}\right)^C$.

Proof: Straightforward.

Definition 2.7

The score function of a CBPFN $C_{BP} = (C, \sigma)$, in which $C = \left(\left[\left(\mu^{+l}_{C_{BP}}, \mu^{+u}_{C_{BP}}\right), \left(\vartheta^{+l}_{C_{BP}}, \vartheta^{+u}_{C_{BP}}\right)\right], \left[\left(\mu^{-l}_{C_{BP}}, \mu^{-u}_{C_{BP}}\right), \left(\vartheta^{-l}_{C_{BP}}, \vartheta^{-u}_{C_{BP}}\right)\right]\right)$ and $\sigma = \left[\mu^{+}_{C_{BP}}, \vartheta^{+}_{C_{BP}}, \mu^{-}_{C_{BP}}, \vartheta^{-}_{C_{BP}}\right]$ is defined as follows:

$$S_C(C_{BP}) = \left(\left(\frac{\mu^{+l}_{C_{BP}}+\mu^{+u}_{C_{BP}}-\mu^{+}_{C_{BP}}}{3}\right)^2 + \left(\frac{\vartheta^{+l}_{C_{BP}}+\vartheta^{+u}_{C_{BP}}-\vartheta^{+}_{C_{BP}}}{3}\right)^2 + \left(\frac{\mu^{-l}_{C_{BP}}+\mu^{-u}_{C_{BP}}-\mu^{-}_{C_{BP}}}{3}\right)^2 + \left(\frac{\vartheta^{-l}_{C_{BP}}+\vartheta^{-u}_{C_{BP}}-\vartheta^{-}_{C_{BP}}}{3}\right)^2\right),$$

where $0 \leq S_C(C_{BP}) \leq 1$

Definition 2.8

Let $C_{BP} = (C, \sigma)$ be a CBPFN, where $C = \left(\left[\left(\mu^{+l}_{C_{BP}}, \mu^{+u}_{C_{BP}}\right), \left(\vartheta^{+l}_{C_{BP}}, \vartheta^{+u}_{C_{BP}}\right)\right], \left[\left(\mu^{-l}_{C_{BP}}, \mu^{-u}_{C_{BP}}\right), \left(\vartheta^{-l}_{C_{BP}}, \vartheta^{-u}_{C_{BP}}\right)\right]\right)$ and $\sigma = \left[\mu^{+}_{C_{BP}}, \vartheta^{+}_{C_{BP}}, \mu^{-}_{C_{BP}}, \vartheta^{-}_{C_{BP}}\right]$.

Then the accuracy function of C_{BP} is defined as follows:

$$A_C(C_{BP}) = \left(\left(\frac{\mu^{+l}_{C_{BP}}+\mu^{+u}_{C_{BP}}-\mu^{+}_{C_{BP}}}{3}\right)^2 - \left(\frac{\vartheta^{+l}_{C_{BP}}+\vartheta^{+u}_{C_{BP}}-\vartheta^{+}_{C_{BP}}}{3}\right)^2 + \left(\frac{\mu^{-l}_{C_{BP}}+\mu^{-u}_{C_{BP}}-\mu^{-}_{C_{BP}}}{3}\right)^2 - \left(\frac{\vartheta^{-l}_{C_{BP}}+\vartheta^{-u}_{C_{BP}}-\vartheta^{-}_{C_{BP}}}{3}\right)^2\right),$$

where $-1 \leq A_C(C_{BP}) \leq 1$.

IV Cubic Bipolar Pythagorean Fuzzy Topology under $\mathbb{R}$-order

In this section, we introduce the concept of a $\mathbb{R}$-cubic bipolar Pythagorean fuzzy topology ($\mathbb{R}$-CBPFT) or a cubic bipolar Pythagorean fuzzy topology with $\mathbb{R}$-order.

Definition 4.1

Let $\mathscr{M}$ be a non-empty universal set and $\mathbb{C}_{BP}(\mathcal{M})$ to be the accumulation of all CBPFSs in $\mathscr{M}$. If the collection $\tau_{\mathbb{R}C_{BP}}$ containing CBPFSs satisfies the following conditions, it is termed as a **$\mathbb{R}$-cubic bipolar Pythagorean fuzzy topology ($\mathbb{R}$-CBPFT),**

(i) ${}^{ll}C_{BP}, {}^{lu}C_{BP}, {}^{ul}C_{BP}, {}^{uu}C_{BP}, \overline{{}^{ll}C_{BP}}, \overline{{}^{lu}C_{BP}}, \overline{{}^{ul}C_{BP}}$ and $\overline{{}^{uu}C_{BP}} \in \tau_{\mathbb{R}C_{BP}}$.

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(ii) If $(\mathbb{R}C_{BP})_i \in \tau_{\mathbb{R}C_{BP}} \forall i$ then $\cup_{\mathbb{R}}(\mathbb{R}C_{BP})_i \in \tau_{\mathbb{R}C_{BP}}$

(iii) If $\mathbb{R}C_{BP_1}, \mathbb{R}C_{BP_2} \in \tau_{\mathbb{R}C_{BP}}$ then $\mathbb{R}C_{BP_1} \cap_{\mathbb{R}} \mathbb{R}C_{BP_2} \in \tau_{\mathbb{R}C_{BP}}$.

Then, the pair $(\mathscr{M}, \tau_{\mathbb{R}C_{BP}})$ is called $\mathbb{R}$-cubic bipolar Pythagorean fuzzy topological space.

Example 4.2

Let $\mathscr{M}$ be a universal set and $\mathbb{RC}_{BP}(\mathcal{M})$ be the assemblage of all $\mathbb{R}$-cubic bipolar Pythagorean fuzzy sets($\mathbb{R}$-CBPFS) in $\mathscr{M}$. Consider $\mathbb{R}$-CBPF subsets of $\mathbb{RC}_{BP}(\mathcal{M})$ given as

$\mathbb{R}C_{BP_1}$ = {[(0, 0), (1, 1)], [(0, 0), (-1, -1)], [0.37, 0.51, -0.42, -0.63]}

$\mathbb{R}C_{BP_1}$ = {[(1, 1), (0, 0)], [(0, 0), (-1, -1)], [0.37, 0.51, -0.42, -0.63]}

$\mathbb{R}C_{BP_1}$ = {[(0, 0), (1, 1)], [(-1, -1), (0, 0)], [0.37, 0.51, -0.42, -0.63]}

$\mathbb{R}C_{BP_1}$ = {[(1, 1), (0, 0)], [(-1, -1), (0, 0)], [0.37, 0.51, -0.42, -0.63]}

The union and intersection with a $\mathbb{R}$-order for the above $\mathbb{R}$-CBPFS are given Tables 1 and 2 respectively.

Table 1 $\mathbb{R}$-Union of $\mathbb{R}$-CBPFS

$\cup_{\mathbb{R}}$	${}^{ll}C_{BP}$	${}^{lu}C_{BP}$	${}^{ul}C_{BP}$	${}^{uu}C_{BP}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
${}^{ll}C_{BP}$	${}^{ll}C_{BP}$	${}^{ll}C_{BP}$	${}^{ul}C_{BP}$	${}^{ul}C_{BP}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
${}^{lu}C_{BP}$	${}^{ll}C_{BP}$	${}^{lu}C_{BP}$	${}^{ul}C_{BP}$	${}^{uu}C_{BP}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
${}^{ul}C_{BP}$	${}^{ul}C_{BP}$	${}^{ul}C_{BP}$	${}^{ul}C_{BP}$	${}^{ul}C_{BP}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$
${}^{uu}C_{BP}$	${}^{ul}C_{BP}$	${}^{uu}C_{BP}$	${}^{ul}C_{BP}$	${}^{uu}C_{BP}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$
$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$
$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$
$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$
$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$
$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_2}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ul}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_2}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{ll}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$
$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\overline{{}^{uu}C_{BP}}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\overline{{}^{lu}C_{BP}}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_4}$

Table 2 $\mathbb{R}$-Intersection of $\mathbb{R}$-CBPFS

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$\cap_{\mathbb{P}}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ul}C_{BP}$	$^{uu}C_{BP}$	$\overline{^{ll}C_{BP}}$	$\overline{^{lu}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\overline{^{uu}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
$^{ll}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{ll}C_{BP}$
$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$
$^{ul}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ul}C_{BP}$	$^{uu}C_{BP}$	$^{uu}C_{BP}$	$^{ul}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{uu}C_{BP}$	$^{ll}C_{BP}$	$^{ul}C_{BP}$
$^{uu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{uu}C_{BP}$	$^{uu}C_{BP}$	$^{uu}C_{BP}$	$^{uu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{uu}C_{BP}$	$^{lu}C_{BP}$	$^{uu}C_{BP}$
$\overline{^{ll}C_{BP}}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{uu}C_{BP}$	$^{uu}C_{BP}$	$\overline{^{ll}C_{BP}}$	$\overline{^{ll}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$
$\overline{^{lu}C_{BP}}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ul}C_{BP}$	$^{uu}C_{BP}$	$\overline{^{ll}C_{BP}}$	$\overline{^{lu}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\overline{^{uu}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$
$\overline{^{ul}C_{BP}}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$\overline{^{ul}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$
$\overline{^{uu}C_{BP}}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$\overline{^{ul}C_{BP}}$	$\overline{^{uu}C_{BP}}$	$\overline{^{ul}C_{BP}}$	$\overline{^{uu}C_{BP}}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_3}$
$\mathbb{R}C_{BP_1}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$
$\mathbb{R}C_{BP_2}$	$^{lu}C_{BP}$	$^{lu}C_{BP}$	$^{uu}C_{BP}$	$^{uu}C_{BP}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$
$\mathbb{R}C_{BP_3}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_3}$
$\mathbb{R}C_{BP_4}$	$^{ll}C_{BP}$	$^{lu}C_{BP}$	$^{ul}C_{BP}$	$^{uu}C_{BP}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_4}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_1}$	$\mathbb{R}C_{BP_2}$	$\mathbb{R}C_{BP_3}$	$\mathbb{R}C_{BP_4}$

Clearly, $\tau^1_{\mathbb{R}C_{BP}} = \left\{ {}^{ll}C_{BP}, {}^{lu}C_{BP}, {}^{ul}C_{BP}, {}^{uu}C_{BP}, \overline{^{ll}C_{BP}}, \overline{^{lu}C_{BP}}, \overline{^{ul}C_{BP}}, \overline{^{uu}C_{BP}} \right\}$

and

$\tau^2_{\mathbb{R}C_{BP}} = \left\{ {}^{ll}C_{BP}, {}^{lu}C_{BP}, {}^{ul}C_{BP}, {}^{uu}C_{BP}, \overline{^{ll}C_{BP}}, \overline{^{lu}C_{BP}}, \overline{^{ul}C_{BP}}, \overline{^{uu}C_{BP}}, \right\}$

are cubic bipolar Pythagorean fuzzy topology with $\mathbb{R}$-order.

Definition 4.3

Let $\mathscr{M}$ be a non-empty universal set and $\tau_{\mathbb{R}C_{BP}} = \{(\mathbb{R}C_{BP})_i / i \in \Lambda\}$ where $(\mathbb{R}C_{BP})_i$ represent the $\mathbb{R}$-cubic bipolar Pythagorean fuzzy subsets of universal set $\mathscr{M}$.

Then, $\tau_{\mathbb{R}C_{BP}}$ is termed as a $\mathbb{R}$-cubic bipolar Pythagorean fuzzy topology on $\mathscr{M}$ and it is the largest $\mathbb{R}$-cubic bipolar Pythagorean fuzzy topology on $\mathscr{M}$ and is called as $\mathbb{R}$-discrete cubic bipolar Pythagorean fuzzy topology.

Definition 4.4

Let $\mathscr{M}$ be a non-empty universal set and $\tau_{\mathbb{R}C_{BP}} = \{ {}^{ll}C_{BP}, {}^{lu}C_{BP}, {}^{ul}C_{BP}, {}^{uu}C_{BP}, \overline{^{ll}\mathrm{C}_{\mathrm{BP}}}, \overline{^{lu}\mathrm{C}_{\mathrm{BP}}}, \overline{^{ul}\mathrm{C}_{\mathrm{BP}}}, \overline{^{uu}\mathrm{C}_{\mathrm{BP}}} \}$ be the collection of $\mathbb{R}$-cubic bipolar Pythagorean fuzzy subsets.

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Then, $\tau_{\mathbb{R}C_{BP}}$ is termed as a ℝ-cubic bipolar Pythagorean fuzzy topology on $\mathscr{M}$ and it is the smallest ℝ-cubic bipolar Pythagorean fuzzy topology on $\mathscr{M}$ and is called as ℝ-indiscrete cubic bipolar Pythagorean fuzzy topology.

Definition 4.5

The elements of ℝ-cubic bipolar Pythagorean fuzzy topology $\tau_{\mathbb{R}C_{BP}}$ is termed as **ℝ-cubic bipolar Pythagorean fuzzy open sets ℝ-CBPFOSs** in ($\mathscr{M}$, $\tau_{\mathbb{R}C_{BP}}$).

Theorem 4.6

If ($\mathscr{M}$, $\tau_{\mathbb{R}C_{BP}}$) is any ℝ-cubic bipolar Pythagorean fuzzy topological space. Then,

(i) ${}^{ll}C_{BP}$, ${}^{lu}C_{BP}$, ${}^{ul}C_{BP}$, ${}^{uu}C_{BP}$, $\overline{{}^{ll}C_{BP}}$, $\overline{{}^{lu}C_{BP}}$, $\overline{{}^{ul}C_{BP}}$ and $\overline{{}^{uu}C_{BP}}$ are ℝ-CBPFOSs

(ii) The ℝ-union of any number of ℝ -CBPFOSs is ℝ-CBPFOS.

(iii) The ℝ -intersection of finite number of ℝ-CBPFOSs is ℝ-CBPFOS.

Proof: The proof is trivial.

Definition 4.7

The complement of elements of ℝ -cubic bipolar Pythagorean fuzzy open sets is termed as **ℝ - cubic bipolar Pythagorean fuzzy closed sets ℝ-CBPFCSs** in ($\mathscr{M}$, $\tau_{\mathbb{R}C_{BP}}$).

Theorem 4.8

If ($\mathscr{M}$, $\tau_{\mathbb{R}C_{BP}}$) is any ℝ-cubic bipolar Pythagorean fuzzy topological space. Then,

(i) ${}^{ll}C_{BP}$, ${}^{lu}C_{BP}$, ${}^{ul}C_{BP}$, ${}^{uu}C_{BP}$, $\overline{{}^{ll}C_{BP}}$, $\overline{{}^{lu}C_{BP}}$, $\overline{{}^{ul}C_{BP}}$ and $\overline{{}^{uu}C_{BP}}$ are ℝ-CBPFCSs

(ii) The ℝ-intersection of any number of ℝ-CBPFCSs is ℝ-CBPFCS.

(iii) The ℝ-union of finite number of ℝ-CBPFCSs is ℝ-CBPFCS.

Proof: The proof is trivial.

Definition 4.9

The ℝ-cubic bipolar Pythagorean fuzzy sets which are ℝ-CBPFOSs and ℝ-CBPFCSs, are termed as ℝ-cubic bipolar Pythagorean fuzzy clopen sets in ($\mathscr{M}$, $\tau_{\mathbb{R}C_{BP}}$).

Proposition 4.1

(i) For every $\tau_{\mathbb{R}C_{BP}}$, ${}^{ll}C_{BP}$, ${}^{lu}C_{BP}$, ${}^{ul}C_{BP}$, ${}^{uu}C_{BP}$, $\overline{{}^{ll}C_{BP}}$, $\overline{{}^{lu}C_{BP}}$, $\overline{{}^{ul}C_{BP}}$ and $\overline{{}^{uu}C_{BP}}$ are ℝ- cubic bipolar Pythagorean fuzzy clopen sets.

(ii) For discrete ℝ- cubic bipolar Pythagorean fuzzy topology, all the ℝ- cubic bipolar Pythagorean fuzzy subsets of $\mathscr{M}$ are ℝ- cubic bipolar Pythagorean fuzzy clopen sets.

(iii) For indiscrete ℝ- cubic bipolar Pythagorean fuzzy topology, ${}^{ll}C_{BP}$, ${}^{lu}C_{BP}$, ${}^{ul}C_{BP}$, ${}^{uu}C_{BP}$, $\overline{{}^{ll}C_{BP}}$, $\overline{{}^{lu}C_{BP}}$, $\overline{{}^{ul}C_{BP}}$ and $\overline{{}^{uu}C_{BP}}$ are only ℝ- cubic bipolar Pythagorean fuzzy clopen sets.

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4.1 Subspace of ℝ -CBPFT

Definition 4.7

Let $(\mathscr{M}, \tau_{\mathbb{R}C_{BP}\mathcal{M}})$ be a ℝ -CBPFT. If $\mathbb{Y} \subseteq \mathscr{M}$, then the collection

$$\tau_{\mathbb{R}C_{BP}\mathbb{Y}} = \{ \mathbb{R}C_{BP}(\mathcal{M}) \cap_{\mathbb{R}} \mathbb{Y} \,/\, \mathbb{R}C_{BP}(\mathcal{M}) \in \tau_{\mathbb{R}C_{BP}\mathcal{M}} \}$$

is a topology on $\mathbb{Y}$ and is called as the ℝ **- cubic bipolar Pythagorean fuzzy subspace topology.** Then, $(\mathbb{Y}, \tau_{\mathbb{R}C_{BP}\mathbb{Y}})$ is called a ℝ **- cubic bipolar Pythagorean fuzzy subspace of** $(\mathscr{M}, \tau_{\mathbb{R}C_{BP}\mathcal{M}})$.

V Algorithm of the Proposed CBPFS – Integrated Entropy – VIKOR – COPRAS Method

In the recommended decision-making process, the use of CBPFS scoring function, entropy weighting technique, VIKOR approach, and COPRAS approach would be involved to rank the available alternatives in the presence of uncertainty. Firstly, the entropy technique will be applied to calculate the objective criteria weights, and then they will be utilized in VIKOR and COPRAS techniques for ranking the alternatives.

5.1 Algorithm: CBPFS - Entropy Weighting Method

Let A = $\{A_1, A_2, \ldots, A_m\}$ be the set of alternatives and C = $\{C_1, C_2, \ldots, C_n\}$ be the set of criteria. Let the CBPFS decision matrix be

$$D = [c_{ij}]_{mxn}$$

where c_{ij} denotes the CBPFN corresponding to alternative A_i under criterion C_j.

1. Construction of the CBPFS decision matrix

The decision matrix is

$$D = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & & c_{2n} \\ \vdots & & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

2. Calculation of the score value

For every CBPFN C_{ij}, calculate its score $s_{ij} = S_C(C_{BP})$ using the score function,

$$S_C(C_{BP}) = \left(\left(\frac{\mu_{C_{BP}}^{+l} + \mu_{C_{BP}}^{+u} - \mu_{C_{BP}}^{+}}{3}\right)^2 + \left(\frac{\vartheta_{C_{BP}}^{+l} + \vartheta_{C_{BP}}^{+u} - \vartheta_{C_{BP}}^{+}}{3}\right)^2 + \left(\frac{\mu_{C_{BP}}^{-l} + \mu_{C_{BP}}^{-u} - \mu_{C_{BP}}^{-}}{3}\right)^2 + \left(\frac{\vartheta_{C_{BP}}^{-l} + \vartheta_{C_{BP}}^{-u} - \vartheta_{C_{BP}}^{-}}{3}\right)^2\right)$$

Consequently, the CBPFS decision matrix is transformed into the score matrix S=$[s_{ij}]_{mxn}$.

3. Normalizing the score matrix

For a Beneficial criterion

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A larger score represents better performance. $P_{ij} = \frac{s_{ij}}{\sum_{j=1}^{5} s_{ij}}$.

For a non-beneficial criterion
A smaller score represents better performance. First transform the score, $s'_{ij} = 1 - s_{ij}$. Then normalize, $P_{ij} = \frac{{s'_{ij}}_{ij}}{\sum_{i=1}^{5} s'_{ij}}$ Thus, $\sum_{j=1}^{5} p_{ij} = 1$. The normalized matrix is P = $\left[p_{ij}\right]_{mxn}$.

4. Determine the entropy of each criterion

For each criterion C_j, calculate $E_j = -\frac{1}{lnm} \sum_{j=1}^{5} p_{ij} ln\, p_{ij}$ where 0ln0 = 0. Therefore, $0 \leq E_j \leq 1$.

The value E_j represents the degree of uncertainty or uniformity associated with criterion C_j.

- Larger E_j: less discriminating information.
- Smaller E_j: greater discriminating information.

5. Calculation of the degree of diversification

The degree of diversification of criterion C_j is d_j = 1−E_j. Hence, 0 ≤ d_j ≤ 1. A larger d_j indicates that criterion C_j provides greater useful information for distinguishing the alternatives.

6. Determine the entropy weights

The objective weight of criterion C_j is $w_j = \frac{d_j}{\sum_{j=1}^{5} d_j}$.

7. Determine the final weight vector:

The resulting criterion weights constitute the final weight vector W= $(w_1, w_2, \ldots. w_n)$ where 0 ≤ w_j ≤ 1 and $\sum_{j=1}^{5} w_j = 1$.

5.2 Algorithm: Proposed CBPFS-VIKOR Method

Let {A_1, A_2, …, A_m} denote the set of alternatives and C = {C_1, C_2, …, C_n} denote the set of criteria. Let D = $\left[C_{ij}\right]_{mxn}$ the

CBPFS decision matrix, and let W= $(w_1, w_2, \ldots. w_n)$ be the weight vector of the criteria calculated using the CBPFS entropy method, where $\sum_{j=1}^{5} w_j = 1$.

1. Construction of the CBPFS Decision Matrix

Evaluate the alternative A_i with respect to criterion C_j using a CBPFN c_{ij}. This gives the CBPFS decision matrix

$$D = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & & c_{2n} \\ \vdots & & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

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2. Calculation of the CBPFS Score Matrix

Calculate $s_{ij} = S_C(C_{BP})$ using the proposed CBPFS score function for i=1, 2, …, m and j=1, 2, …, n. Hence,

the score matrix S=$[s_{ij}]_{mxn}$.

3. Identification of the positive and negative ideal solutions

If the criterion C_j is beneficial, $f_j^* = \max_i s_{ij}, f_j^- = \min_i s_{ij}$.

If the criterion C_j is non-beneficial, $f_j^* = \min_i s_{ij}, f_j^- = \max_i s_{ij}$.

Here, f_j^* represents the positive ideal value and f_j^- represents the negative ideal value.

4. Calculation the normalized distance

For a beneficial criterion, $d_{ij} = \frac{f_j^* - s_{ij}}{f_j^* - f_j^-}$

and for a non-beneficial criterion, $d_{ij} = \frac{s_{ij} - f_j^*}{f_j^- - f_j^*}$

5. Computation of the weighted normalized distance

Using the criteria weights obtained through entropy, the weighted normalized distance, $v_{ij} = w_j d_{ij}$

6. Calculation of the group utility measure

For each alternative A_i, $U_i = \sum_{j=1}^{4} w_j d_{ij}$ where U_i represents the **overall distance from the ideal solution**. A smaller U_i indicates better performance.

7. Calculation of the regret measure of the individual

For each alternative, $R_i = \max_j w_j d_{ij}$. Here, R_i represents the **maximum individual criterion regret**. A smaller R_i is the better.

8. Determine the extreme values

Compute $U^* = \min_i U_i, U^- = \max_i U_i$ and $R^* = \min_i R_i$, $R^- = \max_i R_i$

9. Calculation of the VIKOR index

The VIKOR index of alternative Ai is $Q_i = v\frac{U_i - U^*}{U^- - U^*} + (1 - v)\frac{R_i - R^*}{R^- - R^*}$ where $0 \leq v \leq 1$. Generally,

v = 0.5 is used.

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10. Rank the alternatives

Alternatives are ranked in the **ascending order of Q_i**: $Q_1 < Q_2 < \cdots < Q_m$. Hence, $A^{(1)} > A^{(2)} > \cdots > A^{(m)}$.

The alternative with the **minimum Q_i** is considered the preferred alternative.

11. Verification of the compromise solution

Let us consider $A^{(1)}$ and $A^{(2)}$ are the alternatives ranking first and second with respect to Q_i respectively. The acceptable advantage criterion is $Q(A^{(2)}) - Q(A^{(1)}) \geq \frac{1}{m-1}$. The acceptable stability criterion is the $A^{(1)}$ should rank first also with respect to either U_i or R_i

If both criteria satisfy, then $A^{(1)}$ is considered as the **VIKOR compromise solution**.

5.3 Algorithm: Proposed CBPFS-COPRAS Method

Let $\{A_1, A_2, \ldots, A_m\}$ denote the set of alternatives and $C=\{C_1, C_2, \ldots, C_n\}$ denote the set of criteria. Let D = $[C_{ij}]_{mxn}$ the

CBPFS decision matrix, and let W= $(w_1, w_2, \ldots . w_n)$ be the weight vector of the criteria calculated using the CBPFS entropy method, where $\sum_{j=1}^{5} w_j = 1$.

1. Construction of the CBPFS Decision Matrix

Evaluate the alternative A_i with respect to criterion C_j using a CBPFN C_{ij}. This gives the CBPFS decision matrix

$$D = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & & c_{2n} \\ \vdots & & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

2. Calculation of the CBPFS Score Matrix

Calculate $s_{ij} = S_C(C_{BP})$ using the proposed CBPFS score function for i=1, 2, …, m and j=1, 2, …, n. Hence,

the score matrix S=$[s_{ij}]_{mxn}$.

3. Normalize the score matrix

For a Beneficial criterion

A larger score represents better performance. $p_{ij} = \frac{s_{ij}}{\sum_{i=1}^{5} s_{ij}}$.

For a non-beneficial criterion

A smaller score represents better performance. First transform the score, $s'_{ij} = 1 - s_{ij}$. Then normalize, $P_{ij} = \frac{s'_{ij}{}_{ij}}{\sum_{i=1}^{5} s'_{ij}}$ Thus, $\sum_{j=1}^{5} p_{ij} = 1$. The normalized matrix is P = $[p_{ij}]_{mxn}$.

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4. Construct the weighted normalized matrix

Using the entropy-derived weights, $y_{ij}=w_j\, p_{ij}$. Hence, $Y=\left[y_{ij}\right]_{mxn}$..

5. Calculate the beneficial criterion sum

For each alternative A_i, calculate $B_i = \sum_{j\epsilon c_k}\left[y_{ij}\right]$ where c_k is beneficial criteria. Here, larger B_i indicates better performance.

6. Calculate the non-beneficial criterion sum

For each alternative A_i, calculate $N_i = \sum_{j\epsilon c_l}\left[y_{ij}\right]$ where c_l is non-beneficial criteria. Here, smaller N_i is preferred.

7. Calculate the relative significance

The relative significance of alternative A_i is calculated as $Q_i = B_i + \frac{\sum_{i=1}^{5} N_i}{N_i \sum_{i=1}^{5}\frac{1}{N_i}}$ where $N_i>0$. A larger Q_i indicates a better alternative.

8. Calculate the utility degree

Let Q_{max}= Maximum Q_i. The utility degree is $V_i = \frac{Qi}{Qmax} \times 100$. The alternative with $V_i=100$ has the highest relative utility.

9. Rank the alternatives

Rank the alternatives in descending order of Q_i or V_i: $Q_1 > Q_2 > \cdots > Q_m$. Thus, $A_1 > A_2 > \cdots > A_m$.

The alternative with the maximum Q_i is selected as the best alternative.

VI Numerical Example

Comprehensive Clinical Case Study: CBPFS-Based Cancer Treatment Decision-Making

The medical field of oncology requires some decisions to be made by health specialists. Such elements as tumor stage, tumor grade, biomarker value, patient condition, therapeutic response, and possible risks may be unknown, ambiguous, and even contradictory. As a consequence, the selection of the most appropriate therapy regime is based on the analysis of different clinical factors.

In order to show the practical use of the developed theoretical framework, the suggested decision-making algorithm is applied to the oncology case where different treatment regimens are analyzed by the multidisciplinary tumor board of patients with advanced chronic cancer. The suggested CBPF framework will be used to model the uncertainty in the clinical information and criteria.

Decision-Making Parameters

Alternatives (Treatment Regimens): The board analyzes four different options (m=4):

A_1: Standard systemic Chemotherapy

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A_2: Targeted Radiation therapy

A_3: Advanced Immunotherapy

A_4: combined Modality Palliative Care

Criteria (Clinical Parameters): The treatment protocols are evaluated on the basis of five independent criteria (n=4):

C_1: Treatment Effectiveness (Benefit criterion)

C_2: Patient Response Rate (Benefit criterion)

C_3: Improvement in Quality of Life (Benefit criterion)

C_4: Treatment-Related Side Effects (Non-benefit criterion)

C_5: Treatment Cost (Non-benefit criterion)

6.1 CBPFS - Entropy Weighting Method

Let A={A_1, A_2, …, A_m} be the set of alternatives and C={C_1, C_2, …, C_n} be the set of criteria. Let the

CBPFS decision matrix be D = $[c_{ij}]_{mxn}$ where c_{ij} denotes the CBPFN corresponding to alternative A_i under criterion C_j.

Step 1: Construction of the CBPFS decision matrix

The decision matrix is

	C_1	C_2	C_3	C_4	C_5
A_1	({[0.6, 0.7],[0.2, 0.5], [-0.6, -0.4], [-0.7, -0.5]}, [0.7, 0.4, -0.6, -0.5])	({[0.1, 0.4],[0.5, 0.6], [-0.4, -0.3], [-0.7, -0.5]}, [0.4, 0.6, -0.3, -0.5])	({[0.3, 0.5],[0.4, 0.7], [-0.6, -0.5], [-0.8, -0.5]}, [0.7, 0.6, -0.4, -0.5])	({[0.4, 0.7],[0.3, 0.5], [-0.7, -0.5], [-0.5, -0.3]}, [0.7, 0.4, -0.8, -0.3])	({[0.6, 0.7],[0.3, 0.4], [-0.6, -0.3], [-0.7, -0.2]}, [0.4, 0.3, -0.2, -0.1])
A_2	({[0.2, 0.6],[0.4, 0.7], [-0.5, -0.7], [-0.6, -0.4]}, [0.9, 0.3, -0.4, -0.6])	({[0.5, 0.6],[0.4, 0.7], [-0.6, -0.5], [-0.8, -0.6]}, [0.8, 0.5, -0.7, -0.6])	({[0.6, 0.7],[0.3, 0.5], [-0.4, -0.3], [-0.5, -0.3]}, [0.8, 0.5, -0.3, -0.4})	({[0.5, 0.6],[0.3, 0.7], [-0.8, -0.7], [-0.6, -0.4]}, [0.7, 0.5,-0.4, -0.5])	({[0.4, 0.7],[0.3, 0.5], [-0.5, -0.2], [-0.6, -0.4]}, [0.7, 0.6, -0.5, -0.4])
A_3	({[0.6, 0.7],[0.5, 0.4], [-0.7, -0.5], [-0.4,-0.5]}, [0.4, 0.3, -0.9. -0.4])	({[0.3, 0.5],[0.2, 0.4],[-0.1, -0.3], [-0.7, -0.6]}, [0.3, 0.5, -0.8, -0.4])	({[0.4, 0.5],[0.3, 0.6], [-0.6, -0.2], [-0.7, -0.4]}, [0.8, 0.6, -0.7, -0.5])	({[0.2, 0.3],[0.4, 0.5], [-0.4, -0.8], [-0.6, -0.2]}, [0.4, 0.4, -0.9, -0.1])	({[0.4, 0.6],[0.3, 0.7], [-0.7, -0.5], [-0.6, -0.4]}, [0.8, 0.6, -0.5, -0.3])

RCHUB: JOURNAL OF COMPUTATIONAL INTELLIGENCE SCIENCE AND ENGINEERING (JCISE)
VOL: 2, ISSUE 3, JULY - SEPTEMBER 2026
RCHUB PUBLISHER
ISSN: 3107-8168
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A₄	({[0.4, 0.7],[0.1, 0.4], [-0.9,-0.3], [-0.4,-0.3]}, [0.8, 0.5, -0.9,-0.1])	({[0.5, 0.6], [0.3, 0.6],[-0.7, -0.2], [-0.4, -0.4]}, [0.7, 0.4, -0.5, -0.7])	({[0.1, 0.4],[0.2, 0.4], [-0.8, -0.3], [-0.6, -0.5]}, [0.8, 0.1, -0.9, -0.1])	({[0.7, 0.7],[0.2, 0.5], [-0.8, -0.7], [-0.5, -0.4]}, [0.5, 0.6, -0.4, -0.2])	({[0.6, 0.7],[0.3, 0.5], [-0.6, -0.4], [-0.7, -0.5]}, [0.5, 0.4, -0.6, -0.5])

Step 2: Calculation of the score value

For every CBPFN C_{ij}, calculate its score $s_{ij} = S_C(C_{BP})$ using the score function,

$$S_C(C_{BP}) = \left(\left(\frac{\mu_{C_{BP}}^{+l} + \mu_{C_{BP}}^{+u} - \mu_{C_{BP}}^{+}}{3}\right)^2 + \left(\frac{\vartheta_{C_{BP}}^{+l} + \vartheta_{C_{BP}}^{+u} - \vartheta_{C_{BP}}^{+}}{3}\right)^2 + \left(\frac{\mu_{C_{BP}}^{-l} + \mu_{C_{BP}}^{-u} - \mu_{C_{BP}}^{-}}{3}\right)^2 + \left(\frac{\vartheta_{C_{BP}}^{-l} + \vartheta_{C_{BP}}^{-u} - \vartheta_{C_{BP}}^{-}}{3}\right)^2\right)$$

Now, the CBPFS decision matrix is transformed into the score matrix S=$[s_{ij}]_{mxn}$.

SCORE VALUE					
S_{ij}	**C₁**	**C₂**	**C₃**	**C₄**	**C₅**
A₁	0.3800	0.3033	0.4100	0.2300	0.5933
A₂	0.2967	0.3767	0.2133	0.56333	0.1800
A₃	0.3967	0.1167	0.0567	0.2000	0.3400
A₄	0.0867	0.1733	0.0800	0.6667	0.4767

Step 3: Normalizing the score matrix

For a Beneficial criterion

A larger score represents better performance. $P_{ij} = \frac{s_{ij}}{\sum_{j=1}^{5} s_{ij}}$.

For a non-beneficial criterion

A smaller score represents better performance. First transform the score, $s'_{ij} = 1 - s_{ij}$. Then normalize, $P_{ij} = \frac{s'_{ij_{ij}}}{\sum_{i=1}^{5} s'_{ij}}$ Thus, $\sum_{j=1}^{5} p_{ij} = 1$. The normalized matrix is P = $[p_{ij}]_{mxn}$.

p_{ij}	**C₁**	**C₂**	**C₃**	**C₄**	**C₅**
A₁	0.3276	0.3127	0.5395	0.3291	0.1687
A₂	0.2558	0.3883	0.2807	0.1866	0.3403
A₃	0.3420	0.1203	0.0746	0.3419	0.2739
A₄	0.0747	0.1787	0.1053	0.1425	0.2172

Step 4: Determine the entropy of each criterion

For each criterion C_j, calculate $E_j == -\frac{1}{lnm} \sum_{j=1}^{5} p_{ij} ln\, p_{ij}$ where 0ln0 = 0. Therefore, $0 \leq E_j \leq 1$.

The value E_j represents the degree of uncertainty or uniformity associated with criterion C_j.

- Larger E_j: less discriminating information.

VOL: 2, ISSUE 3, JULY - SEPTEMBER 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/3/009

- Smaller E_j: greater discriminating information.

p_{ij} *ln p_{ij}	C_1	C_2	C_3	C_4	C_5
A_1	-0.1588	-0.1579	-0.1446	-0.1588	-0.1304
A_2	-0.1515	-0.1595	-0.1549	-0.1361	-0.1593
A_3	-0.1594	-0.1106	-0.0841	-0.1594	-0.154
A_4	-0.0842	-0.1336	-0.1029	-0.1206	-0.144
E_j	0.91977	0.93292	0.80799	0.95475	0.97626

Step 5: Calculation of the degree of diversification

The degree of diversification of criterion C_j is $d_j = 1 - E_j$. Hence, $0 \le d_j \le 1$. A larger d_j indicates that criterion C_j provides greater useful information for distinguishing the alternatives.

$d_j = 1 - E_j$	0.08023	0.06708	0.19201	0.04525	0.02374

Step 6: Determine the entropy weights

The objective weight of criterion C_j is $w_j = \frac{d_j}{\sum_{j=1}^{5} d_j}$.

w_j	0.20862	0.17442	0.49929	0.11767	0.06173

Step 7: Determine the final weight vector:

The resulting criterion weights constitute the final weight vector as

$$W = (0.20862, 0.17442, 0.49929, 0.11767, 0.06173), \text{ where } \sum_{j=1}^{5} w_j = 1.$$

6.2 Proposed CBPFS-VIKOR Method

Let $\{A_1, A_2, \ldots, A_m\}$ denote the set of alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ denote the set of criteria. Let $D = [C_{ij}]_{mxn}$ be the same CBPFS decision matrix as in the CBPFS - Entropy, and let W= (0.20862, 0.17442, 0.49929, 0.11767,

0.06173) be the weight vector of the criteria calculated using the CBPFS entropy method, where $\sum_{j=1}^{5} w_j = 1$.

Step 1: Calculation of the CBPFS Score Matrix

Calculate $s_{ij} = S_C(C_{BP})$ using the proposed CBPFS score function for i=1, 2, …, m and j=1, 2, …, n. Hence,

the score matrix $S = [s_{ij}]_{mxn}$ is

w_j	0.20862	0.17442	0.49929	0.11767	0.06173
	C_1	C_2	C_3	C_4	C_5

RCHUB: JOURNAL OF COMPUTATIONAL INTELLIGENCE SCIENCE AND ENGINEERING (JCISE)
VOL: 2, ISSUE 3, JULY - SEPTEMBER 2026
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A1	0.3800	0.3033	0.4100	0.2300	0.5933
A2	0.2967	0.3767	0.2133	0.5633	0.1800
A3	0.3967	0.1167	0.0567	0.2000	0.3400
A4	0.0867	0.1733	0.0800	0.6667	0.4767

Step 2: Identification of the positive and negative ideal solutions

If the criterion C_j is beneficial, $f_j^* = \max_i s_{ij}, f_j^- = \min_i s_{ij}$.

If the criterion C_j is non-beneficial, $f_j^* = \min_i s_{ij}, f_j^- = \max_i s_{ij}$.

Here, f_j^* represents the positive ideal value and f_j^- represents the negative ideal value.

f_j^*	0.3967	0.3767	0.4100	0.2000	0.1800
f_j^-	0.0867	0.1167	0.0567	0.6667	0.5933

Step 3: Calculate the normalized distance

For a beneficial criterion, $d_{ij} = \frac{f_j^* - s_{ij}}{f_j^* - f_j^-}$

For a non-beneficial criterion, $d_{ij} = \frac{s_{ij} - f_j^*}{f_j^- - f_j^*}$

d_{ij}	**C1**	**C2**	**C3**	**C4**	**C5**
A1	0.0538	0.2821	0.0000	0.0643	1.0000
A2	0.3226	0.0000	0.5566	0.7786	0.0000
A3	0.0000	1.0000	1.0000	0.0000	0.3871
A4	1.0000	0.7821	0.9340	1.0000	0.7177

Step 4: Computation of the weighted normalized distance

Using the criteria weights obtained through entropy, the weighted normalized distance, $v_{ij} = w_j d_{ij}$

v_{ij}	**C1**	**C2**	**C3**	**C4**	**C5**
A1	0.0112	0.0492	0.0000	0.0076	0.0617
A2	0.0673	0.0000	0.2779	0.0916	0.0000
A3	0.0000	0.1744	0.4993	0.0000	0.0239
A4	0.2086	0.1364	0.4663	0.1177	0.0443

Step 5: Calculation of the group utility measure

For each alternative A_i, $U_i = \sum_{j=1}^{4} w_j d_{ij}$ where U_i represents the **overall distance from the ideal solution**. A smaller U_i indicates better performance.

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/3/009

U^*	0.1297	R^*	0.0617
U^-	0.9733	R^-	0.4663

U_i	Rank
0.1297	1
0.4368	2
0.6976	3
0.9733	4

Step 6: Calculation of the regret measure of the individual

For each alternative, $R_i = \max_j w_j d_{ij}$. Here, R_i represents the **maximum individual criterion regret**. A smaller R_i is the better.

R_i
0.0617
0.2779
0.4993
0.4663

Step 7: Determine the extreme values

Compute $U^* = \min_i U_i, U^- = \max_i U_i$ and $R^* = \min_i R_i$, $R^- = \max_i R_i$.

Step 8: Calculation of the VIKOR index and rank the alternatives

The VIKOR index of alternative Ai is

$$Q_i = v\frac{U_i - U^*}{U^- - U^*} + (1 - v)\frac{R_i - R^*}{R^- - R^*} \text{ where } 0 \le v \le 1.$$

Generally, v = 0.5 is used. Alternatives are ranked in the **ascending order of Q_i**: $Q_1 < Q_2 < \cdots < Q_m$. The alternative with the **minimum Q_i** is considered the preferred alternative.

Q_i	Rank
0	1
4.4917	2
8.7733	3
10	4

Step 9: Verification of the compromise solution

Let us consider $A^{(1)} = \mathbf{A_1}$ and $A^{(2)} = \mathbf{A_2}$ are the alternatives ranking first and second with respect to Q_i respectively. The

acceptable advantage criterion is

ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/3/009

$$Q(A_2) - Q(A_1) = 4.4917 \geq \frac{1}{4-1} = 0.333.$$

The acceptable stability criterion is the A_1 should rank first also with respect to either U_i or R_i.If both criteria satisfy, then

A_1 is considered as the **VIKOR compromise solution**.

6.3 Proposed CBPFS-COPRAS Method

Let $\{A_1, A_2, \ldots, A_m\}$ denote the set of alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ denote the set of criteria. Let $D = [C_{ij}]_{mxn}$ be

the same CBPFS decision matrix as in the CBPFS - Entropy, and let W= (0.20862, 0.17442, 0.49929, 0.11767,

0.06173) be the weight vector of the criteria calculated using the CBPFS entropy method, where $\sum_{j=1}^{5} w_j = 1$.

Step 1: Calculation of the CBPFS Score Matrix

Calculate $s_{ij} = S_C(C_{BP})$ using the proposed CBPFS score function for i=1, 2, …, m and j=1, 2, …, n. Hence,

the score matrix $S=[s_{ij}]_{mxn}$ is

w_j	0.20862	0.17442	0.49929	0.11767	0.06173
	C_1	**C_2**	**C_3**	**C_4**	**C_5**
A_1	0.3800	0.3033	0.4100	0.2300	0.5933
A_2	0.2967	0.3767	0.2133	0.5633	0.1800
A_3	0.3967	0.1167	0.0567	0.2000	0.3400
A_4	0.0867	0.1733	0.0800	0.6667	0.4767

Step 2: Normalizing the score matrix

For a Beneficial criterion A larger score represents better performance. $p_{ij} = \frac{s_{ij}}{\sum_{i=1}^{5} s_{ij}}$.

For a non-beneficial criterion A smaller score represents better performance. First transform the score, $s'_{ij} = 1 - s_{ij}$. Then normalize, $P_{ij} = \frac{s'_{ij}{}_{ij}}{\sum_{i=1}^{5} s'_{ij}}$ Thus, $\sum_{j=1}^{5} p_{ij} = 1$. The normalized matrix is $P = [p_{ij}]_{mxn}$.

p_{ij}	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**
A_1	0.3276	0.3127	0.5395	0.3291	0.1687
A_2	0.2557	0.3883	0.2807	0.1866	0.3402

VOL: 2, ISSUE 3, JULY - SEPTEMBER 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/3/009

A_3	0.3420	0.1203	0.0746	0.3419	0.2739
A_4	0.0747	0.1787	0.1053	0.1425	0.2172

Step 3: Construction of the weighted normalized matrix

Using the entropy-derived weights, $y_{ij} = w_j p_{ij}$. Hence, $Y=[y_{ij}]_{mxn}$.

y_{ij}	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**
A_1	0.0683	0.0545	0.2694	0.0387	0.0104
A_2	0.0534	0.0677	0.1402	0.0220	0.0210
A_3	0.0713	0.0210	0.0372	0.0402	0.0169
A_4	0.0156	0.0312	0.0526	0.0168	0.0134

Step 4: Calculation of the beneficial criterion sum and non-beneficial criterion sum

For each alternative A_i, calculate $B_i = \sum_{j\epsilon c_k}[y_{ij}]$ where c_k is beneficial criteria. Here, larger B_i indicates better performance. For each alternative A_i, calculate $N_i = \sum_{j\epsilon c_l}[y_{ij}]$ where c_l is non-beneficial criteria. Here, smaller N_i is preferred.

Benefit B_i	Non-benefit N_i
0.3922	0.0491
0.2612	0.0430
0.1295	0.0571
0.0993	0.0302

Step 5: Calculation of the relative significance

The relative significance of alternative A_i is calculated as $Q_i = B_i + \frac{\sum_{i=1}^{5} N_i}{N_i \sum_{i=1}^{5} \frac{1}{N_i}}$ where $N_i>0$. A larger Q_i indicates a better alternative.

$\sum \frac{1}{N_i}$	$\frac{\sum N_i}{N_i \sum \frac{1}{N_i}}$
20.3514	0.0387
23.2759	0.0443
17.5025	0.0333
33.1484	0.0631

VOL: 2, ISSUE 3, JULY - SEPTEMBER 2026
RCHUB PUBLISHER
ISSN: 3107-8168
DOI: https://doi.org/10.65725/JCISE/2/3/009

Step 5: Calculation of the utility degree and ranking the alternatives

Let Q_{max}= Maximum Q_i. The utility degree is $V_i = \frac{Qi}{Qmax} \times 100$. The alternative with V_i=100 has the highest relative utility.

Rank the alternatives in descending order of Q_i or V_i: $Q_1 > Q_2 > \cdots > Q_m$. Thus, $A_1 > A_2 > \cdots > A_m$.

The alternative with the maximum Q_i is selected as the best alternative.

Qi	Vi	Vi *100	Rank
0.4310	**1**	**100**	**1**
0.3055	**0.70894**	**70.8938**	**2**
0.1629	**0.37788**	**37.7876**	**3**
0.1624	**0.3768**	**37.6804**	**4**

6.4 Comparative Analysis of CBPFS-Entropy-VIKOR and CBPFS-Entropy-COPRAS

To validate the reliability and robustness of the proposed **CBPFS-based MCDM framework**, the obtained results were compared using two different ranking approaches, namely **CBPFS-Entropy-VIKOR** and **CBPFS-Entropy-COPRAS**. Both approaches were applied to the same CBPFS decision matrix, using the same CBPFS score function and the same criterion weights obtained through the entropy method.

The results obtained from both approaches produce the **same ranking of the alternatives**, given by $A_1 > A_2 > A_3 > A_4$. Thus, A_3 is identified as the most preferred alternative, while A_4 occupies the lowest position in both approaches. The identical ranking demonstrates the **stability and robustness** of the proposed CBPFS-based decision-making framework. Although VIKOR and COPRAS follow different aggregation principles, both methods identify A3 as the optimal alternative. VIKOR determines a compromise solution by considering the overall group utility and the maximum individual regret, whereas COPRAS evaluates alternatives according to their relative significance with respect to beneficial and non-beneficial criteria.

6.5 Future Work

The proposed hybrid decision-making approach based on CBPFS can be further developed in many directions. Other topological characteristics of CBPFSs considering (R)-order relation can be developed and analyzed in future works together with their relationships from the theoretical point of view. Future works may include the extension of the proposed entropy-VIKOR-COPRAS decision-making approach by using other objective weighting methods and MCDM approaches, for instance, CRITIC, TOPSIS, WASPAS, and MAIRCA. Moreover, future studies can extend CBPFSs to **interval-valued, hesitant, spherical, and (q)-rung CBPFSs** that can help in modeling of more complicated uncertainties. Proposed methodology can also be used in many other healthcare issues, such as **disease diagnosis, treatment choice, medical resource allocation, and healthcare planning**.

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VOL: 2, ISSUE 3, JULY - SEPTEMBER 2026
RCHUB PUBLISHER
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