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A GRAPH-THEORETICAL AND MATHEMATICAL ANALYSIS OF MUSICAL NOTES, RHYTHM, AND SEVEN-KEY TRANSITIONS

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Abstract

This study examines seven supplied musical lines through rhythmic timing and a directed graph representation of note-to-note transitions. The musical vocabulary consists of seven keys—A, B, C♯, D, E, F♯, and G— which form the vertices of the network. The repeat indications in Lines 5 and 6 are expanded as specified, yielding 113 note occurrences and 106 consecutive transitions. Each transition is counted between adjacent notes within its own numbered line; no transition is added between separate numbered lines. The resulting seven-by-seven transition-count matrix is examined through global normalization, in-strength, out-strength, directed betweenness, outward closeness, PageRank, self-loops, reciprocal pairs, directed three-node cycles, graph density, selected subgraphs, and spectral radius. The results identify F♯ as the leading vertex in incoming strength, directed betweenness, outward closeness, and PageRank, while F♯ and G share the highest outgoing strength. The temporal analysis applies the beat-duration relation 60/BPM and, where a group is established to divide one beat equally, the note-duration relation 60/(BPM × n). The reference tempos of 60 BPM and 120 BPM are compared. Tempo-based duration is kept distinct from transition frequency so that each quantity retains its own mathematical interpretation. The study provides a reproducible framework for describing the selected musical passage through note succession, timing, and network structure.

Keywords: musical graph theory; directed weighted graph; transition matrix; note sequence; rhythmic duration; centrality; PageRank; seven-key network

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Introduction

Music is an organized temporal art in which pitch, rhythm, duration, repetition, and succession work together to create recognizable structures. Although a musical composition is experienced through sound, its ordered events can also be represented by mathematical objects. A sequence of notes can be treated as an ordered set of observations, and the movement from one note to the next can be represented by a directed relation. Graph theory therefore provides a formal language for describing selected aspects of musical organization without replacing the musical identity of the source material.

The present study examines seven supplied musical lines. The note vocabulary consists of seven distinct keys: A, B, C♯, D, E, F♯, and G. These keys form the vertex set of the transition network. Each consecutive movement from one note to another is recorded as a directed transition. Direction is essential: a movement from A to B is not automatically equivalent to a movement from B to A. Each named pitch, including a sharped pitch, is treated as one musical event so that the note sequence and graph vertices use the same vocabulary.

The seven lines contain different numbers of notes and rhythmic groupings. Lines 5 and 6 carry repeat indications, and each is expanded twice in the event sequence used for counting. Within each repeated line, the end of the first occurrence is followed by the beginning of the next occurrence; that repeat boundary is included as a consecutive transition. Transitions between separate numbered lines are not added unless they are explicitly part of the same line sequence. Under these conventions, the expanded data contain 113 note occurrences and 106 consecutive transitions.

Two complementary descriptions are developed. The temporal description uses beats per minute and the rhythmic grouping shown in the supplied notation. For a tempo of BPM beats per minute, one beat lasts 60/BPM seconds. When a group of n notes is established to divide one beat equally, each note is assigned 60/(BPM × n) seconds. This condition matters because note spacing on a printed page, by itself, is not sufficient evidence of equal note duration. The reference tempos of 60 BPM and 120 BPM provide a consistent basis for comparing beat and note durations.

The supplied timeline places the opening passage and the seven numbered lines in temporal order: the opening interval is 0:00–0:05, Lines 1–6 extend through 1:18, and Line 7 continues to the stated end of the selected passage. These intervals locate the musical material but do not determine transition frequency. Tempo controls elapsed time, whereas the transition matrix records which movements occur and how frequently they occur.

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The graph description records note-to-note succession, repeated transitions, and the connectivity of the seven-key system. A directed weighted graph is constructed by treating the seven keys as vertices and each observed consecutive movement as an edge. The transition-count matrix records the number of occurrences of every ordered pair. A binary adjacency matrix is also used to distinguish the existence of an edge from its frequency. These representations answer different questions and are not interchangeable.

The count matrix is globally normalized by dividing each entry by the total of 106 transitions. This expresses each ordered transition as a share of all observed movements. Row normalization is performed separately to obtain the transition probabilities used for PageRank. Weighted strength measures are derived from transition counts, while reachability, shortest paths, and graph density are evaluated on the binary support graph. The distinction between these mathematical representations is maintained throughout the analysis.

The network is examined through in-strength and out-strength, directed betweenness, outward closeness, PageRank, self-loops, reciprocal two-node pairs, directed three-node cycles, graph density, selected subgraphs, and spectral radius. The five-key chain is used as an illustrative subnetwork, and its values are distinguished from those calculated on the complete seven-key graph. This prevents local examples from being mistaken for results of the full network.

The main objective is to establish a traceable route from the supplied musical observations to quantitative network measures. The analysis proceeds from the note sequences and timing assumptions to the transition-count matrix and its derived measures. The resulting network is interpreted through several complementary properties rather than through a single numerical indicator. A key with high transition strength need not have the same role under shortest-path centrality or PageRank.

The technical body that follows retains the original Chapter 2 analysis, including its note transcriptions, timing formulas, BPM comparisons, transition construction and verification, normalized matrices, strength and centrality calculations, PageRank, cycle analysis, density, subgraph classification, spectral result, consolidated measures, and final results. The following sections therefore provide the detailed calculations supporting the scope and methodological conventions established in this introduction.

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2. Literature Review

Quantitative approaches to music provide a way to examine musical organization through measurable features rather than relying only on descriptive listening. Mauch et al. (2015) analysed a large collection of popular recordings using music information-retrieval methods and identified measurable changes in harmonic and timbral properties over time. Their work demonstrates the value of systematic, data-driven musical analysis, although its focus is broad historical comparison rather than a single passage represented as a
seven-vertex directed transition graph.

A related line of research represents musical material as sequences of discrete events and studies relationships between successive events. Serra et al. (2012) examined musical pitch-transition networks and used network properties such as shortest paths and clustering to investigate structural patterns in contemporary popular music. This provides a close methodological precedent for treating musical transitions as network links. The present study applies a more focused, passage-level construction: each of the seven supplied keys is a vertex, each adjacent note pair is a directed edge, and repeated transitions are retained as edge weights.

The mathematical interpretation of such networks relies on established graph and network concepts. Newman (2010) provides a general framework for analysing network structure, including paths, centrality, connectivity, and weighted relationships. These concepts inform the distinction made in this paper between transition frequency, binary reachability, and shortest-path measures. In particular, the study calculates strength from the weighted transition counts, while path-based centralities are evaluated on the binary support graph.

PageRank was introduced by Brin and Page (1998) as a recursive ranking method based on the structure of directed links. Although their application concerned hyperlinks rather than music, the mathematical principle can be transferred to a directed transition network by constructing a row-stochastic transition matrix and applying a damping factor. The present study follows this general principle, using the observed outgoing transition counts to form the transition probabilities and reporting the seven-key PageRank distribution separately from global count normalization.

Taken together, these studies establish relevant foundations for quantitative music analysis, musical transition networks, general network measures, and PageRank. The contribution of the present work is narrower and data-specific: it documents a reproducible graph model for seven supplied musical lines, explicitly records the repeat treatment, distinguishes the 113 note occurrences from the 106 within-line transitions, and reports several complementary network measures for the resulting seven-key graph. The literature reviewed here supports the analytical methods; it does not establish that this exact musical passage or album has never been studied before.

Scope note: The cited works are methodological and contextual references. They are not presented as prior analyses of the exact seven-line dataset used in this paper.

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2.2. Musical Notes

A musical note sequence is an ordered arrangement of note events. In the present representation, a note name is one token even when it contains a sharp sign. Thus $F\sharp$ is one musical key, not two separate events, and $C\sharp$ is treated in the same way. The seven-key vocabulary is therefore

$$V = \{A, B, C\sharp, D, E, F\sharp, G\}.$$

The note sequence is retained in its original order because transition direction is obtained from consecutive tokens. For instance, the local sequence $G, F\sharp, A$ produces the directed transitions $G \rightarrow F\sharp$ and $F\sharp \rightarrow A$. Reversing the order would produce different transitions and a different directed graph.

The seven lines contain different numbers of notes and different rhythmic groupings. Some note groups repeat. Lines 5 and 6 carry a repeat indication, and each is expanded twice in the event sequence used for counting. Within each repeated line, the end of the first occurrence is followed by the beginning of the next occurrence; that repeat boundary is included as a consecutive transition. Transitions between separate numbered lines are not added unless they are explicitly part of the same line sequence.

Definition 2.1. A musical note sequence is a finite ordered list $X = (x_1, x_2, \ldots, x_m)$, where each x_k belongs to the note vocabulary V.

Example 2.1. For the opening line, the token sequence is $B, B, A, G, F\sharp, A, G, F\sharp, E, E$. Its order defines nine consecutive transitions.

Explanation. Tokenization is performed by reading each named pitch as one event. This avoids incorrectly splitting sharped notes into separate symbols.

Key Point. The note sequence is an ordered data set, not merely a list of distinct pitches.

Interpretation. Repeated notes remain meaningful because they may produce self-loops or increase the frequency of an existing directed edge.

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2.2.1. Musical Notes

The seven musical lines are recorded below. The original pitch vocabulary is preserved, and the repeat indications for Lines 5 and 6 are written explicitly. Spaces separate note groups in the supplied transcription; they do not change the rule that each pitch token is counted as one event.

Line 1: BB A G F#AGF# EE

Line 2: B EE B EE B E EF#GA BB A G F#AF# EE

Line 3: EF# GF# AG F#G F# G F# G F# G F# G F#D

Line 4: EF# GF# AG F#G F#G F#G F#G F#G F#

Line 5: E DE F#GA A GF# GAB $\times 2$

Line 6: B A BE D C# B A $\times 2$

Line 7: GF# EF# G A B

After the repeat marks are expanded, the event counts are 10, 21, 18, 17, 24, 16 and 7 for Lines 1–7, respectively. Their sum is

$$10 + 21 + 18 + 17 + 24 + 16 + 7 = 113.$$

For a line containing m_r consecutive events, the number of within-line transitions is $m_r - 1$. The repeat joins in Lines 5 and 6 are included in those expanded sequences. Consequently, the transition counts by line are 9, 20, 17, 16, 23, 15 and 6.

$$9 + 20 + 17 + 16 + 23 + 15 + 6 = 106.$$

Definition 2.2. A note occurrence is one appearance of a pitch token in the expanded ordered sequence.

Example 2.2. A repeated line containing 12 notes and played twice contributes 24 note occurrences and 23 consecutive transitions when the repeat boundary is included.

Key Point. The expanded data contain 113 note occurrences and 106 consecutive transitions.

Interpretation. The difference between these totals is seven because each of the seven numbered lines has one starting event with no preceding event inside that line.

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2.3 The Core Conversion Formulas

Beats per minute (BPM) specifies the number of beats occurring in one minute. The duration of one beat is therefore the number of seconds in a minute divided by the number of beats in that minute:

$$t_{\text{beat}} = \frac{60}{\text{BPM}} \quad \text{seconds.}$$

When a displayed rhythmic group occupies one beat and contains n equally subdivided notes, the duration assigned to each note is

$$t_{\text{note}} = \frac{t_{\text{beat}}}{n} = \frac{60}{\text{BPM}\,n} \quad \text{seconds.}$$

The equal-subdivision condition is important. The formula applies only when the score or the stated rhythmic grouping establishes that the notes divide one beat equally. Note spacing on a printed page, by itself, is not sufficient evidence of equal note duration.

For a single note occupying one beat, $n = 1$, so the note duration is $60/\text{BPM}$. For a two-note group occupying one beat, each note receives half a beat, so the duration is $30/\text{BPM}$. A three-note equal subdivision gives $20/\text{BPM}$ seconds per note, and a four-note equal subdivision gives $15/\text{BPM}$ seconds per note.

Definition 2.3. The beat-duration formula converts a tempo value in BPM into elapsed seconds per beat.

Definition 2.4. The equal-subdivision formula assigns one equal share of a beat to each note in a group of n notes.

Example 2.3. At 60 BPM, one beat lasts $60/60 = 1$ second. If that beat is divided equally into four notes, each note lasts $1/4 = 0.25$ second.

Key Point. The general duration rule is $60/(\text{BPM} \times n)$, subject to the one-beat and equal-subdivision assumptions.

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2.3.1 The Core Conversion Formulas

The formula can be evaluated directly at the two reference tempos used in this study.

$$t_{\text{beat}}(60) = \frac{60}{60} = 1.00 \text{ s},$$

$$t_{\text{beat}}(120) = \frac{60}{120} = 0.50 \text{ s}.$$

At 60 BPM, a one-beat group lasts 1.00 second. Two equal notes within that beat each last 0.50 second; three equal notes each last approximately 0.3333 second; and four equal notes each last 0.25 second. At 120 BPM, the corresponding durations are 0.50, 0.25, approximately 0.1667 and 0.125 second.

Number of equal notes n	Formula per note	60 BPM	120 BPM
1	60/(BPM)	1.0000 s	0.5000 s
2	60/(2 BPM)	0.5000 s	0.2500 s
3	60/(3 BPM)	0.3333 s	0.1667 s
4	60/(4 BPM)	0.2500 s	0.1250 s

Example 2.4. For a two-note group at 120 BPM, the beat lasts 0.5 second. Equal subdivision gives $0.5/2 = 0.25$ second per note.

Explanation. The duration of a beat decreases when BPM increases. If the grouping size remains unchanged, each equally subdivided note also becomes shorter in the same proportion.

Key Point. At 120 BPM, every corresponding beat-based duration is half its value at 60 BPM.

Interpretation. These values provide a consistent time scale for the selected rhythmic groups. They do not, on their own, specify the weight of a graph edge; the graph-count model is defined separately.

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2.4 Beats Per Minute (BPM)

BPM is a tempo measure. A tempo of 60 BPM means that 60 beats occur in one minute, while 120 BPM means that 120 beats occur in one minute. The time per beat is inversely proportional to the tempo. Thus, doubling the BPM halves the duration of one beat.

The two reference tempos are used to compare the same musical grouping under different timing conditions. At 60 BPM, the beat period is one second. At 120 BPM, the beat period is half a second. If the score indicates that a group of four notes occupies one beat, the group lasts one second at 60 BPM and half a second at 120 BPM; each of its four equally divided notes lasts 0.25 and 0.125 second, respectively.

Tempo	Beat duration	Beats in 10 seconds
60 BPM	1.0000 s	10
120 BPM	0.5000 s	20
180 BPM	0.3333 s	30

The 180 BPM row is included as an arithmetic reference only. The main comparisons in the present analysis use 60 BPM and 120 BPM. The tempo value determines the clock-time scale, but it does not alter the order of the notes. A transition such as $F\sharp \rightarrow G$ remains the same directed transition at either tempo; only its elapsed-time interpretation changes when a valid rhythmic duration has been assigned.

Definition 2.5. BPM is the number of beats performed per minute.

Example 2.5. At 120 BPM, $60/120 = 0.5$ second per beat.

Key Point. BPM governs temporal duration, not the identity or direction of a note transition.

Interpretation. Separating tempo from network topology allows the same graph structure to be compared under more than one performance speed.

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2.5 Note Duration Chart

A note-duration chart summarizes the time assigned to different equal subdivisions at a selected tempo. The grouping sizes shown below correspond to one, two, three or four equal notes within one beat. The chart is a conversion reference: the actual rhythmic value in the score determines which row applies to a particular note group.

Grouping	Example from the supplied lines	60 BPM	120 BPM
One note per beat	*B, A, G*	1.0000 s	0.5000 s
Two equal notes per beat	*BB, EE, GF♯*	0.5000 s each	0.2500 s each
Three equal notes per beat	*GAB*	0.3333 s each	0.1667 s each
Four equal notes per beat	*F♯AGF♯, EF♯GA*	0.2500 s each	0.1250 s each

The examples identify note groups used in the transcription. Their durations are interpreted using the equal-subdivision rule only when the notation establishes that the group occupies one beat. A group of four notes is not automatically a four-note subdivision merely because four symbols appear close together; its rhythmic value must be determined from the score.

For a group containing n equally spaced subdivisions, the total group duration is one beat and the individual note duration is $60/(\text{BPM}\ n)$. If a group occupies more than one beat, the total group value must be included before dividing among its notes. This distinction prevents a group-size formula from being applied outside its stated assumptions.

Example 2.6. A four-note group occupying one beat at 60 BPM has a total duration of 1 second and an individual-note duration of $1/4 = 0.25$ second.

Key Point. A duration chart is valid only when the beat-value and subdivision of each group are known.

Interpretation. The chart connects the written rhythmic organization to measurable time while preserving the distinction between rhythmic evidence and graph connectivity.

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2.6 Rhythm

Rhythm describes the temporal organization of musical events. The supplied timeline identifies an opening signature followed by seven numbered lines. The intervals are recorded below in minutes and seconds. The end point of Line 7 is not numerically specified in the supplied timing description and is therefore retained as "end".

Segment	Start	End	Supplied note
Opening signature	0:00	0:05	Introductory passage
Line 1	0:05	0:11	Opening line
Line 2	0:11	0:24	–
Line 3	0:24	0:35	–
Line 4	0:35	0:46	–
Line 5	0:46	1:02	Repeated twice
Line 6	1:02	1:18	Repeated twice
Line 7	1:18	End	Final line

The line intervals provide a temporal location for the note sequences. They do not change the transition-count definition. In particular, a line may contain many note events over a short interval, and two lines with similar durations may have different numbers of transitions. Temporal duration and transition frequency are therefore recorded as separate variables.

Definition 2.6. A rhythmic timeline is an ordered set of start and end times associated with musical segments.

Example 2.7. Line 1 occupies the supplied interval from 0:05 to 0:11, while Line 2 occupies 0:11 to 0:24.

Explanation. The timeline positions each line within the selected passage and makes it possible to discuss temporal order independently of graph connectivity.

Key Point. The timeline specifies when the line occurs; the transition sequence specifies how its notes follow one another.

Interpretation. The two representations complement one another but should not be merged without an explicitly defined time-weighting rule.

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2.7.1 Seven-Key Musical Transition Network

A directed transition network is formed by taking the seven distinct musical keys as vertices and recording each consecutive note movement as an edge. Let

$$V = \{A, B, C\sharp, D, E, F\sharp, G\}.$$

For two keys $i, j \in V$, the ordered pair $i \to j$ is present when at least one consecutive occurrence of i followed by j appears in the expanded line sequences. Repeated occurrences of the same ordered pair are counted rather than discarded.

Definition 2.7. A musical transition network is a directed graph $G = (V, E)$, where each vertex represents one distinct key and each directed edge represents a consecutive movement between two keys.

Definition 2.8. A vertex is a member of the set V and represents one of the seven musical keys.

Definition 2.9. A directed edge (i, j) records a movement from source key i to destination key j.

Definition 2.10. The transition count C_{ij} is the number of times the ordered pair $i \to j$ occurs in the expanded musical sequence.

The network has two levels of information. The first is the binary question of whether a transition occurs at least once. The second is its frequency, recorded by the non-negative integer C_{ij}. Thus, $C_{ij} = 0$ means that the transition is absent, while a positive value records its number of occurrences.

Example 2.8. The entry $C_{A,B} = 7$ means that the transition $A \to B$ occurs seven times in the expanded data.

Key Point. The graph is directed because the musical order of the source and destination notes is retained.

Interpretation. The count matrix is reproducible from the line sequences and supplies the common numerical basis for the network measures that follow.

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2.7.2 Transition-Count Construction

The transition matrix is constructed by scanning each expanded line from its first note to its last note. For every adjacent pair (x_k, x_{k+1}), the corresponding entry is increased by one:

$$C_{ij} = \sum_{r=1}^{7} \sum_{k=1}^{m_r - 1} \mathbf{1}\{x_{r,k} = i \text{ and } x_{r,k+1} = j\},$$

where m_r is the number of events in expanded Line r, and $\mathbf{1}\{\cdot\}$ is an indicator equal to one when the condition is satisfied and zero otherwise.

The order of operations is important. First, each sharped pitch is tokenized as one key. Second, Lines 5 and 6 are expanded according to their repeat marks. Third, the sequence of each numbered line is scanned for adjacent pairs. Fourth, every occurrence is accumulated in the row corresponding to the source key and the column corresponding to the destination key.

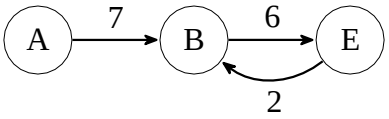


Figure 1: Illustrative directed transitions with their occurrence counts

The figure illustrates how a transition and its reverse are represented separately. A positive count in one direction does not imply that the reverse direction has the same count. The count matrix preserves this distinction for every ordered pair of keys.

Example 2.9. If a line contains the consecutive pattern *A*, *B*, *E*, *B*, the transitions added are $A \rightarrow B$, $B \rightarrow E$, and $E \rightarrow B$.

Key Point. The matrix is obtained by counting adjacent ordered pairs, not by counting only distinct note names.

Interpretation. Repeated musical material increases the corresponding transition count and is therefore retained in the weighted directed network.

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[1.] The Seven-by-Seven Transition Matrix

Rows of the transition matrix represent source keys, and columns represent destination keys. The order of both rows and columns is fixed as *A*, *B*, *C♯*, *D*, *E*, *F♯*, *G*. The complete transition-count matrix is

$$C = \begin{bmatrix} 2 & 7 & 0 & 0 & 0 & 1 & 7 \\ 6 & 2 & 0 & 0 & 6 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 4 & 5 & 6 & 0 \\ 4 & 0 & 0 & 1 & 3 & 0 & 18 \\ 6 & 0 & 0 & 0 & 0 & 20 & 0 \end{bmatrix}.$$

From \ To	A	B	C#	D	E	F#	G
A	2	7	0	0	0	1	7
B	6	2	0	0	6	0	0
C#	0	2	0	0	0	0	0
D	0	0	2	0	2	0	0
E	0	2	0	4	5	6	0
F#	4	0	0	1	3	0	18
G	6	0	0	0	0	20	0

Every entry is a non-negative integer. The diagonal entries represent self-transitions: $A \to A$ occurs twice, $B \to B$ occurs twice, and $E \to E$ occurs five times. The zero diagonal entries for *C♯*, *D*, and *F♯* indicate that no self-transition for those keys occurs in the expanded line data.

Example 2.10. The row for *F♯* records four transitions to *A*, one to *D*, three to *E*, and eighteen to *G*. Its row total is 26.

Key Point. The matrix contains all 106 consecutive transition occurrences in the seven-line data.

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[2.] Matrix Verification and Edge List

The sum of all entries in the transition matrix gives the total number of consecutive transitions:

$$\sum_{i \in V} \sum_{j \in V} C_{ij} = 106.$$

The row totals and column totals are calculated independently. A row total gives the number of transitions leaving a key; a column total gives the number entering a key.

Key	Row total (out count)	Column total (in count)
A	17	18
B	14	13
C#	2	2
D	4	5
E	17	16
F#	26	27
G	26	25
Total	106	106

The equality of the two grand totals follows from the fact that every transition has exactly one source and exactly one destination. This is a useful consistency check for any directed transition matrix. The active ordered pairs are the entries greater than zero. There are 20 such pairs, including three diagonal pairs. Excluding self-loops, there are 17 active non-loop ordered pairs.

The active edges are $A \to A$, $A \to B$, $A \to F\#$, $A \to G$; $B \to A$, $B \to B$, $B \to E$; $C\# \to B$; $D \to C\#$, $D \to E$; $E \to B$, $E \to D$, $E \to E$, $E \to F\#$; $F\# \to A$, $F\# \to D$, $F\# \to E$, $F\# \to G$; and $G \to A$, $G \to F\#$.

Key Point. The row and column totals both sum to 106, and the matrix has 20 distinct active directed key-pairs.

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2.8 Graph-Theoretical Interpretation

A directed graph represents the seven musical keys as vertices and the active transitions as oriented edges. A weighted version assigns the transition count C_{ij} to the edge from i to j. The binary adjacency matrix is obtained by converting every positive count to one:

$$B_{ij} = \begin{cases} 1, & C_{ij} > 0, \\ 0, & C_{ij} = 0. \end{cases}$$

The binary matrix describes the existence of a transition, while the count matrix records how many times it occurs. Both representations are useful, but they answer different questions. The binary graph is appropriate for questions about paths, reachability and unweighted centrality. The count matrix is appropriate for frequency-based strength and weighted PageRank.

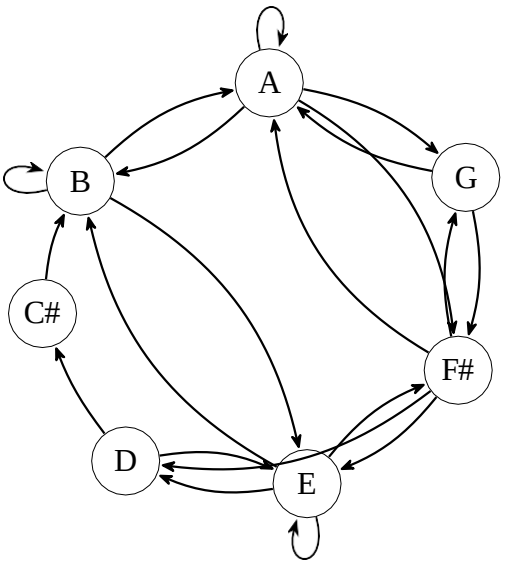


Figure 2: Directed seven-key transition structure, including its active self-loops

Explanation. An arrow records a permitted direction of movement. Reciprocal arrows appear only when both directions occur in the source data.

Key Point. A directed edge is not interchangeable with its reverse.

Interpretation. The graph converts the ordered musical data into a network whose local and global properties can be evaluated mathematically.

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2.9 Paths and Directed Connectivity

A path is an ordered sequence of vertices in which each consecutive pair is connected by a directed edge. For example, the sequence

$$E \to F\sharp \to G \to A \to B$$

is a directed path in the supplied transition network. It illustrates how a note can be reached through a chain of intermediate keys. A path is valid only when every directed edge in the sequence has a positive transition count.

A closed directed path that begins and ends at the same vertex is a cycle. A self-loop is a cycle of length one. A two-node reciprocal cycle uses two vertices and two opposite directed edges. Longer cycles involve three or more distinct vertices before returning to the starting point.

The seven-key graph is strongly connected: for every ordered pair of vertices, a directed path exists from the first vertex to the second. This means that every key is reachable from every other key, although the shortest-path lengths need not be equal. Strong connectivity is a property of the directed support graph and does not depend on the magnitude of the transition counts once positive entries are treated as present edges.

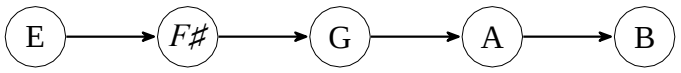


Figure 3: Five-key directed path used as an illustrative subnetwork

Definition 2.11. A directed path from *s* to *t* is a vertex sequence beginning at *s*, ending at *t*, and following the direction of every edge.

Key Point. Reachability concerns the existence of a path, whereas shortest-path distance concerns the minimum number of edges in such a path.

Interpretation. The path example is a selected five-key subnetwork; seven-key results are calculated separately from the full matrix.

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2.10 Mathematical Unified Model

The complete count-based model is expressed as a weighted directed graph

$$G_2 = (V, E, C),$$

where V is the seven-key vertex set, E is the set of active directed key-pairs, and $C = [C_{ij}]$ is the transition-count matrix. The note sequence defines the entries of C, so the model is reproducible from the ordered musical data.

For a global frequency normalization, define

$$W = \frac{C}{T}, \qquad T = \sum_{i,j} C_{ij} = 106.$$

The entry $W_{ij} = C_{ij}/106$ is the proportion of all observed transitions represented by the ordered pair $i \to j$. Therefore,

$$\sum_{i,j} W_{ij} = 1.$$

The sum of row i in W is the global count-based out-strength of vertex i; the sum of column j is the global count-based in-strength of vertex j. Since all entries are divided by the same total, the total out-strength and total in-strength are both one.

This count-based normalization is distinct from duration normalization. A duration-weighted graph would require a rule assigning a time value to every transition, for example the duration of the source note or an inter-onset interval. Those alternatives are not equivalent. Accordingly, the model in this section uses transition frequency and does not relabel the count matrix as a duration matrix.

Definition 2.12. The global normalized transition weight is $W_{ij} = C_{ij}/106$.

Example 2.12. The transition $A \to B$ occurs seven times, so $W_{A,B} = 7/106 \approx 0.06604$.

Key Point. The count-normalized matrix is globally normalized, with all entries summing to one.

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2.11 Global Normalized Transition Matrix

Dividing each transition count by 106 produces the global normalized matrix. Values are displayed to five decimal places; the exact definition remains $W = C/106$.

$$W \approx \begin{bmatrix} 0.01887 & 0.06604 & 0 & 0 & 0 & 0.00943 & 0.06604 \\ 0.05660 & 0.01887 & 0 & 0 & 0.05660 & 0 & 0 \\ 0 & 0.01887 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01887 & 0 & 0.01887 & 0 & 0 \\ 0 & 0.01887 & 0 & 0.03774 & 0.04717 & 0.05660 & 0 \\ 0.03774 & 0 & 0 & 0.00943 & 0.02830 & 0 & 0.16981 \\ 0.05660 & 0 & 0 & 0 & 0 & 0.18868 & 0 \end{bmatrix}.$$

The table form of this matrix can be read in the same way as the count matrix: each row is a source key and each column is a destination key. For example, the normalized entry for $G \to F\#$ is $20/106 \approx 0.18868$, the largest individual transition weight. The normalized entry for $F\# \to G$ is $18/106 \approx 0.16981$.

A globally normalized matrix is useful for comparing the relative contribution of each ordered pair to the complete transition data. It is not row-stochastic: individual row sums are generally not equal to one. The row-stochastic matrix needed for PageRank is defined separately by dividing each row by its own row total.

Example 2.13. For $F\# \to G$, $W_{F\#,G} = 18/106 = 0.169811$.

Explanation. Keeping global normalization and row normalization distinct prevents the same symbol from being used for two different probability models.

Key Point. The global matrix W measures each edge as a share of all 106 transitions.

Interpretation. Large normalized values identify transitions that contribute strongly to the total observed frequency.

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2.12 Line-Wise Event and Transition Totals

Each line is counted after applying its repeat indication. The number of transitions in a line is the number of events minus one because each line is treated as its own ordered sequence. For Lines 5 and 6, the two repeated occurrences are joined at the repeat boundary, so that the boundary transition is included.

Line	Repeat factor	Note occurrences	Consecutive transitions
1	1	10	9
2	1	21	20
3	1	18	17
4	1	17	16
5	2	24	23
6	2	16	15
7	1	7	6
Total	–	113	106

The total event count is obtained by adding the event count of each expanded line. The total transition count is obtained by adding the consecutive-pair count of each line. No transition is added between the end of one numbered line and the beginning of the next numbered line. This convention keeps the line boundaries consistent with the supplied note-line organization.

$$N_{\text{events}} = 113, \qquad N_{\text{transitions}} = \sum_{r=1} (m_r - 1) = 106.$$

Definition 2.13. The line-wise transition total is the number of adjacent note pairs within an expanded numbered line.

Example 2.14. Expanded Line 5 contains 24 notes and therefore contributes 23 consecutive transitions.

Key Point. The event and transition totals provide an independent check on the sum of the matrix entries.

Interpretation. The line-wise table links the original musical transcription to the aggregated seven-by-seven transition matrix.

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2.13 Verification of the Count Basis

The count basis can be verified in two independent ways. First, the expanded note sequences can be scanned directly, adding one to the relevant matrix cell for each consecutive pair. Second, the row totals of the resulting matrix can be summed. Both procedures give 106 transitions.

The relation between event and transition counts is particularly useful. Within a line containing m notes, there are exactly $m - 1$ consecutive pairs. A first note has no preceding note within that line, while every later note has exactly one immediate predecessor. Across seven separately treated lines, the total number of transitions is therefore the total event count minus seven:

$$113 - 7 = 106.$$

The row totals provide the same result:

$$17 + 14 + 2 + 4 + 17 + 26 + 26 = 106.$$

The column totals also provide 106:

$$18 + 13 + 2 + 5 + 16 + 27 + 25 = 106.$$

The equality of these totals confirms that the transition records have been counted consistently. It does not depend on the choice of tempo because transition frequency is a property of the ordered sequence.

Example 2.15. The row for key *A* totals 17, showing that 17 of the 106 observed transitions leave *A*. The column for *A* totals 18, showing that 18 transitions enter *A*.

Explanation. The event-count check, row-sum check and column-sum check test different aspects of the same transition data.

Key Point. A valid directed transition matrix must have equal total outgoing and incoming counts.

Interpretation. The three checks agree at 106, supporting the internal consistency of the count-based network representation.

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2.14 Graph Value: In-Strength and Out-Strength

For a directed weighted graph, out-strength measures the sum of weights on edges leaving a vertex, and in-strength measures the sum of weights on edges entering it. With the global normalized matrix $W = C/106$, these quantities are

$$S_i^{\text{out}} = \sum_{j \in V} W_{ij}, \qquad S_i^{\text{in}} = \sum_{j \in V} W_{ji}.$$

Equivalently, the count-based strengths are the corresponding row and column totals divided by 106:

$$S_i^{\text{out}} = \frac{\sum_j C_{ij}}{106}, \qquad S_i^{\text{in}} = \frac{\sum_j C_{ji}}{106}.$$

These measures describe the relative amount of transition activity associated with each key. A high out-strength indicates that many transitions leave the key in the observed sequence. A high in-strength indicates that many transitions arrive at the key. Neither measure, considered alone, describes shortest-path position or recursive importance; those properties are examined separately through centrality and PageRank.

Definition 2.14. The out-strength of a vertex is the sum of its outgoing normalized edge weights.

Definition 2.15. The in-strength of a vertex is the sum of its incoming normalized edge weights.

Example 2.16. The out-strength of *F♯* is $26/106 = 0.24528$, while its in-strength is $27/106 = 0.25472$.

Key Point. Out-strength and in-strength are row and column sums of the globally normalized count matrix.

Interpretation. The values measure participation in transition frequency and sum to one across all vertices for each direction.

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2.15 Count-Based Strength Results

The row and column totals of the transition matrix yield the strength values in the following table. Values are rounded to five decimal places.

Key	Out count	In count	Out-strength	In-strength
A	17	18	0.16038	0.16981
B	14	13	0.13208	0.12264
C#	2	2	0.01887	0.01887
D	4	5	0.03774	0.04717
E	17	16	0.16038	0.15094
F#	26	27	0.24528	0.25472
G	26	25	0.24528	0.23585
Total	106	106	1.00000	1.00000

For example, the outgoing strength of *G* is $26/106 = 0.24528$, and its incoming strength is $25/106 = 0.23585$. The total outgoing and incoming strengths both equal one because every count is divided by the same global total. Small differences in the final displayed digit may arise from rounding; the unrounded fractions are exact.

The largest outgoing count is 26, shared by *F♯* and *G*. The largest incoming count is 27, belonging to *F♯*. Thus the same vertex has the greatest incoming transition frequency, while the outgoing maximum is shared by two vertices.

Example 2.17. For *B*, $S_B^{\text{out}} = 14/106 \approx 0.13208$ and $S_B^{\text{in}} = 13/106 \approx 0.12264$.

Key Point. The outgoing maximum is a tie between *F♯* and *G*; the incoming maximum is *F♯*.

Interpretation. The count-based strengths provide a reproducible measure of transition frequency for the complete seven-key network.

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2.16 Dominant Transition Hub

A transition hub is a vertex with a comparatively high level of transition activity or network importance. In the present count-based model, the hub interpretation is supported by more than one measure rather than by a single quantity.

The outgoing strength table shows that *F♯* and *G* tie for the highest outgoing strength:

$$S^{\text{out}}_{F\sharp} = S^{\text{out}}_{G} = \frac{26}{106} = 0.24528.$$

The highest incoming strength belongs to *F♯*:

$$S^{\text{in}}_{F\sharp} = \frac{27}{106} = 0.25472.$$

The weighted PageRank calculation later in the chapter also assigns the largest score to *F♯*, with approximately 0.21085. These results support identifying *F♯* as the leading transition hub overall, while recognizing that *G* shares the maximum outgoing count.

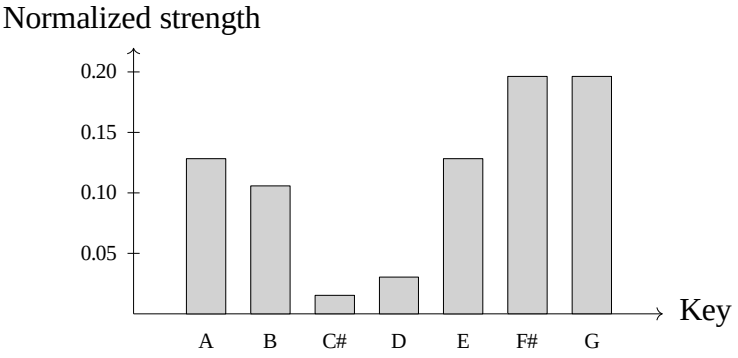


Figure 4: Outgoing count-based strength across the seven musical keys

Key Point. The hub conclusion is based on the joint evidence from incoming strength, outgoing strength and PageRank.

Interpretation. The network gives *F♯* the strongest overall central role, while *G* is equally active in outgoing transition frequency.

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2.17 Betweenness Centrality

Betweenness centrality measures the extent to which a vertex lies on shortest paths between other ordered pairs of vertices. It differs from strength: strength counts direct transition activity, whereas betweenness measures participation as an intermediate point in paths that connect other keys.

Let σ_{st} denote the number of shortest directed paths from s to t, and let $\sigma_{st}(v)$ denote the number of those paths that pass through an intermediate vertex v. The unnormalized directed betweenness is

$$C_B(v) = \sum_{\substack{s \neq v \neq t \\ s \neq t}} \frac{\sigma_{st}(v)}{\sigma_{st}}.$$

For a directed graph with n vertices, normalized betweenness is

$$C^*_B(v) = \frac{C_B(v)}{(n-1)(n-2)}.$$

Here $n = 7$, so the normalization denominator is $6 \times 5 = 30$. Shortest paths are calculated on the binary support graph: a matrix entry greater than zero represents one directed edge, and its numerical frequency is not treated as a path length.

Definition 2.16. Betweenness centrality is the sum of the fractions of shortest directed paths between other vertices that pass through a given vertex.

Example 2.18. If a unique shortest path from *A* to *D* passes through *F♯*, then that ordered pair contributes one to the unnormalized betweenness of *F♯*.

Explanation. If multiple shortest paths exist, the contribution is divided among the paths according to the fraction that passes through the vertex.

Key Point. The complete graph uses 30 as the normalization denominator for directed betweenness.

Interpretation. Betweenness identifies bridge-like positions that may not be obvious from direct transition counts alone.

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2.18 Betweenness Results for the Seven-Key Network

Applying the directed shortest-path definition to the seven-key support graph gives the following normalized values. The graph is strongly connected, so every ordered pair of distinct vertices has a finite directed shortest-path distance.

Key	Normalized betweenness	Relative position
A	0.1500	Intermediate
B	0.2222	Important bridge
C#	0.0278	Low bridge participation
D	0.1667	Intermediate
E	0.2556	Important bridge
F#	0.2778	Highest betweenness
G	0.0000	No shortest-path intermediary role

The largest value belongs to *F♯*, at approximately 0.2778. The next largest values are associated with *E* (0.2556) and *B* (0.2222). The result differs from the five-key illustrative chain considered later because the complete seven-key network contains additional routes through *A*, *B*, *C♯*, *D*, and *E*.

The value zero for *G* does not mean that *G* is absent or unimportant. It means that, under the binary shortest-path calculation, *G* does not occur as an intermediate vertex on shortest paths between other ordered pairs. The vertex still has high transition strength and a high PageR-ank value. This difference demonstrates why several graph measures are needed to describe a network.

Example 2.19. The normalized betweenness of *F♯* is approximately 0.2778, the largest value in the seven-key support graph.

Key Point. The full seven-key betweenness ranking begins $F\sharp > E > B > D > A > C\sharp > G$.

Interpretation. The results identify *F♯* as the most prominent shortest-path intermediary in the complete directed network.

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2.19 Directed Betweenness in the Five-Key Illustrative Chain

The sequence $E \rightarrow F\sharp \rightarrow G \rightarrow A \rightarrow B$ is a five-key path used to illustrate the centrality definitions. It is a selected subnetwork, not the complete seven-key transition graph. The path has five vertices and four directed edges.

For an ordered directed chain, the normalized denominator is

$$(n-1)(n-2) = 4 \times 3 = 12.$$

The intermediate vertex $F\sharp$ lies on the three directed shortest paths from E to G, A, and B, so its normalized directed betweenness is $3/12 = 0.25$. The vertex G lies on four relevant ordered paths, giving $4/12 = 0.3333$. The vertex A lies on three, giving $3/12 = 0.25$. The endpoints have zero betweenness.

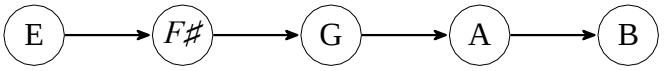


Figure 5: The five-key directed chain used for the illustrative calculation

Key	Intermediate ordered pairs	Directed betweenness
E	0	0.0000
F#	3	0.2500
G	4	0.3333
A	3	0.2500
B	0	0.0000

Key Point. The five-key chain values describe only the displayed path; the seven-key values are reported separately.

Interpretation. The middle vertex G has the greatest directed betweenness in this illustrative chain.

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2.20 Closeness Centrality

Closeness centrality measures how near a vertex is to the other vertices through shortest paths. A small sum of shortest-path distances gives a large closeness value. Because the network is directed, the direction of the distance must be stated.

For outward closeness, the distance is measured from the selected vertex v to every other vertex u:

$$C_C^{\text{out}}(v) = \sum_{u \neq v} \frac{n-1}{d(v,u)},$$

where $d(v, u)$ is the length of the shortest directed path from v to u. With seven vertices, the numerator is six. The denominator is therefore the sum of six directed distances from the selected key to the remaining keys.

The graph is strongly connected, so every outward distance is finite. The same matrix could also be used to calculate inward closeness by measuring paths from the other vertices toward v, but that is a different directed measure. The values in this chapter use the outward convention to match the displayed formula.

Definition 2.17. The outward closeness of a vertex is $(n-1)$ divided by the sum of its shortest directed distances to all other vertices.

Example 2.20. If a vertex has total outward distance 8 in a seven-vertex graph, then its outward closeness is $6/8 = 0.75$.

Explanation. The measure rewards vertices that can reach the rest of the network through relatively short directed paths.

Key Point. The outward direction is explicitly used in the seven-key closeness calculation.

Interpretation. Closeness describes access to the rest of the network, whereas betweenness describes intermediary participation between other vertices.

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2.21 Closeness Calculation

The shortest directed distances are calculated on the binary support graph. For each vertex, the six distances to the other vertices are added, and the numerator six is divided by that sum.

Key	Sum of outward shortest-path distances	Outward closeness
A	10	0.6000
B	11	0.5455
C#	14	0.4286
D	12	0.5000
E	9	0.6667
F#	8	0.7500
G	11	0.5455

For example, the total outward distance from *F♯* is eight. Its closeness is

$$C_C^{\text{out}}(F\sharp) = \frac{6}{8} = 0.75.$$

The smallest total distance is also associated with *F♯*, so it has the largest outward closeness. The value for *E* is $6/9 = 0.6667$, while *A* has $6/10 = 0.6000$. The ordering shows that the network is accessible from several keys, but the distance structure is not uniform.

Example 2.21. For *C♯*, the outward distance total is 14 and the closeness value is $6/14 \approx 0.4286$.

Key Point. The largest outward closeness in the complete seven-key graph is 0.7500 at *F♯*.

Interpretation. The result indicates that *F♯* reaches all other keys with the smallest total number of directed steps.

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2.22 Combined Centrality Interpretation

Betweenness and closeness provide different descriptions of vertex position. Betweenness measures how frequently a vertex is used as an intermediary on shortest paths between other vertices. Outward closeness measures how efficiently a vertex can reach the other vertices. A key may score highly on one measure without having the highest value on the other.

Key	Betweenness	Outward closeness	Combined observation
A	0.1500	0.6000	Intermediate access
B	0.2222	0.5455	Bridge participation
C#	0.0278	0.4286	Low centrality
D	0.1667	0.5000	Intermediate position
E	0.2556	0.6667	Strong intermediary and access
F#	0.2778	0.7500	Highest on both measures
G	0.0000	0.5455	Reachable, not a shortest-path intermediary

The complete-network comparison places *F♯* first in both normalized betweenness and outward closeness. The key *E* is second in betweenness and also has high closeness. The key *G* has zero betweenness in this calculation but retains a high degree of direct transition activity. This illustrates that centrality measures should be interpreted according to their mathematical definitions rather than treated as interchangeable rankings.

The five-key chain gives a different but compatible illustration. In that path, *G* is the middle vertex and has the largest closeness and betweenness values within that five-node subnetwork. The result does not conflict with the complete graph because the two calculations use different vertex sets and different edge structures.

Key Point. The complete seven-key graph identifies *F♯* as the leading vertex under the two directed path measures used here.

Interpretation. Centrality results are meaningful only when the graph scope, edge direction, path convention and normalization are stated.

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2.23 PageRank

PageRank is a recursive measure of vertex importance. A vertex receives importance from the vertices that point to it; the contribution received from each predecessor is divided according to that predecessor's outgoing transition distribution. PageRank therefore depends on both direct connectivity and the importance of the vertices supplying the incoming transitions.

For the seven-key network, let $r_i = \sum_j C_{ij}$ be the outgoing count of vertex i. Define the row-stochastic transition matrix

$$P_{ij} = \frac{C_{ij}}{r_i} \quad (r_i > 0).$$

Each row of P sums to one. With damping factor $d = 0.85$ and $n = 7$ vertices, PageRank satisfies

$$PR(j) = \frac{1-d}{n} + d\sum_{i \in V} PR(i)P_{ij}.$$

The term $(1 - d)/n$ represents the uniform teleportation contribution, while the second term distributes rank through the directed transition structure. Since each vertex in this graph has positive out-count, no dangling-row adjustment is required.

Definition 2.18. PageRank is the stationary importance distribution obtained by repeatedly distributing each vertex's rank over its outgoing transitions and adding a uniform damping term.

Example 2.22. For this analysis, the damping factor is $d = 0.85$ and the uniform contribution per key is $(1 - 0.85)/7 \approx 0.0214286$.

Key Point. PageRank uses the row-stochastic matrix P, not the globally normalized matrix W.

Interpretation. The count-frequency network supplies the transition probabilities, while damping ensures that the rank distribution remains well-defined.

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2.24 PageRank Calculation for Seven Keys

The outgoing totals used to construct P are 17, 14, 2, 4, 17, 26 and 26 for the keys *A, B, C♯, D, E, F♯, G*. For example, the row corresponding to *A* is divided by 17, so its outgoing probabilities are $2/17$, $7/17$, 0, 0, 0, $1/17$, $7/17$. The row corresponding to *F♯* is divided by 26.

The PageRank vector is obtained from the fixed-point equation

$$\boldsymbol{p} = \frac{0.15}{7}\mathbf{1} + 0.85P^{\mathrm{T}}\boldsymbol{p}, \qquad \sum_i p_i = 1.$$

Iterating this equation until successive vectors converge gives the results below.

Key	PageRank ($d = 0.85$)
A	0.16034
B	0.15304
C#	0.04751
D	0.06137
E	0.16526
F#	0.21085
G	0.20163
Total	1.00000

The largest PageRank value is associated with *F♯*, followed by *G* and *E*. The smallest value is associated with *C♯*. The values sum to one, as required for a probability distribution. These values are based on the seven-key transition-count matrix and are distinct from the illustrative five-key chain values.

Example 2.23. The PageRank of *F♯* is approximately 0.21085, meaning that it receives the largest share of the stationary rank distribution.

Key Point. The seven-key PageRank ranking begins *F♯ > G > E > A > B > D > C♯*.

Interpretation. Recursive transition flow emphasizes *F♯* as the most important vertex under the specified damping and count-probability model.

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2.25 PageRank Cases

A simplified degree-based measure and the standard damped PageRank measure can be compared using the five-key undirected path $E - F\sharp - G - A - B$. In the simplified case, rank is proportional to degree. The two endpoints have degree one, and the three internal vertices have degree two. The total degree is eight, giving

$$PR_{\text{simple}}(E) = PR_{\text{simple}}(B) = \frac{1}{8} = 0.125,$$

$$PR_{\text{simple}}(F\sharp) = PR_{\text{simple}}(G) = PR_{\text{simple}}(A) = \frac{2}{8} = 0.250.$$

For the same undirected path, standard PageRank with damping factor 0.85 is obtained by solving the stationary equation using the row-normalized adjacency matrix. The resulting values are approximately 0.13453 for each endpoint, 0.24595 for $F\sharp$ and A, and 0.23905 for G. The damping term slightly increases the endpoint shares and changes the distribution among the internal vertices.

Key	Simplified degree share	PageRank ($d = 0.85$)
E	0.12500	0.13453
F#	0.25000	0.24595
G	0.25000	0.23905
A	0.25000	0.24595
B	0.12500	0.13453

These five-key values illustrate how model choice affects the numerical ranking. They must not be substituted for the seven-key PageRank values calculated from the full transition-count matrix.

Key Point. PageRank values depend on the graph and normalization model used.

Interpretation. A five-key path is useful for explaining the algorithm, while the seven-key matrix supplies the principal network result.

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2.26 Cycle Analysis: Self-Loops

A self-loop is a directed edge that begins and ends at the same vertex. It is represented by a diagonal entry of the transition matrix. In the present matrix, the non-zero diagonal entries are

$$C_{A,A} = 2, \qquad C_{B,B} = 2, \qquad C_{E,E} = 5.$$

There are therefore three distinct self-loop edges, with nine self-loop transition occurrences in total:

$$2 + 2 + 5 = 9.$$

The distinction between the number of self-loop edges and the number of self-loop occurrences is important. The first counts distinct key pairs on the diagonal; the second sums how often those self-transitions appear in the musical sequence.

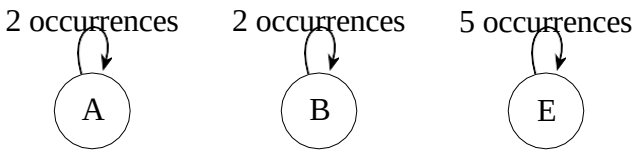


Figure 6: Self-loop transitions at *A*, *B* and *E*

Definition 2.19. A self-loop is an edge $i \to i$ from a vertex to itself.

Example 2.24. The transition $E \to E$ occurs five times and is represented by the diagonal matrix entry $C_{E,E} = 5$.

Key Point. The data contain three distinct self-loop edges and nine self-loop occurrences.

Interpretation. Self-loops record repeated retention of the same musical key between consecutive note events.

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2.27 Two-Node Reciprocal Cycles

A two-node reciprocal cycle occurs when both directions between two distinct vertices are present. For two keys i and j, the required conditions are $C_{ij} > 0$ and $C_{ji} > 0$. Such a pair forms the directed cycle $i \to j \to i$.

The transition matrix contains seven reciprocal pairs:

$$A \leftrightarrow B, \qquad A \leftrightarrow F\sharp, \qquad A \leftrightarrow G,$$
$$B \leftrightarrow E, \qquad D \leftrightarrow E,$$
$$E \leftrightarrow F\sharp, \qquad F\sharp \leftrightarrow G.$$

Each listed pair has positive counts in both directions. For example, $A \to B$ occurs seven times and $B \to A$ occurs six times. The reciprocal structure is therefore present even though the two directional counts are not equal.

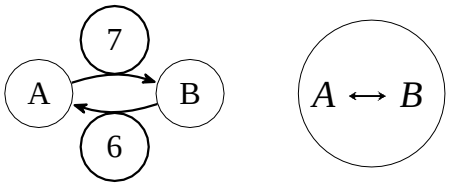


Figure 7: Example of a reciprocal two-node cycle

Definition 2.20. A reciprocal pair is an unordered pair of distinct vertices for which both directed edges are active.

Example 2.25. The pair $E, F\sharp$ is reciprocal because $C_{E,F\sharp} = 6$ and $C_{F\sharp,E} = 3$.

Key Point. Seven reciprocal two-node cycles are supported by the transition-count matrix.

Interpretation. Reciprocal edges show that the network contains return movement between selected pairs of musical keys.

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2.28 Three-Node Cycles

A directed three-node cycle is a closed sequence of three distinct vertices in which each vertex has an outgoing edge to the next and the final vertex returns to the starting vertex. The matrix confirms three directed cycles of length three:

$$A \to G \to F\sharp \to A,$$

$$A \to F\sharp \to G \to A,$$

$$D \to E \to F\sharp \to D.$$

The first two cycles use the same three vertices but opposite orientations. They are distinct directed cycles because the edge order is different. If cycles are grouped only by their underlying set of vertices, the first two describe one three-vertex group; when direction is retained, they are two separate cycles.

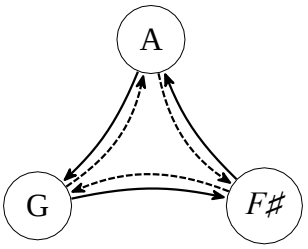


Figure 8: Two directed orientations on the three-key set $\{A, G, F\sharp\}$

The proposed sequence $C\sharp \to F\sharp \to D \to C\sharp$ is not a cycle because the transition $C\sharp \to F\sharp$ is absent. The matrix therefore distinguishes verified cycles from sequences that fail one or more required edges.

Definition 2.21. A directed three-node cycle is a closed directed path containing exactly three distinct vertices.

Key Point. The matrix supports three directed cycles of length three, on two underlying vertex sets.

Interpretation. Preserving direction is essential when identifying cyclic musical structures.

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2.29 Seven-by-Seven Graph Density

Graph density measures the proportion of possible directed edges that are active. The count matrix has 20 distinct active ordered pairs, of which three are self-loops. The remaining 17 are non-loop directed edges.

For a directed graph in which self-loops are allowed, the number of possible ordered pairs is

$$|E_{\max}| = n^2 = 7^2 = 49.$$

The loop-inclusive density is therefore

$$D_{\text{loop}} = \frac{20}{49} = 0.408163 \approx 40.82\%.$$

Under the standard simple directed-graph convention that excludes self-loops, the number of possible edges is

$$|E^{\circ}_{\max}| = n(n-1) = 7 \times 6 = 42,$$

and the non-loop density is

$$D_{\text{no loop}} = \frac{17}{42} = 0.404762 \approx 40.48\%.$$

Convention	Active edges / possible edges	Density
Self-loops included	20/49	0.40816 (40.82%)
Self-loops excluded	17/42	0.40476 (40.48%)

The two values are both valid under their stated conventions and are not interchangeable. The loop-inclusive value counts all 20 positive matrix positions. The loop-excluded value removes the three diagonal positions from both the observed edge set and the set of possible ordered pairs.

Definition 2.22. Directed graph density is the ratio of active directed edges to the number of theoretically possible directed edges under a stated loop convention.

Key Point. The density of the seven-key network is 40.82% when loops are included and 40.48% when loops are excluded.

Interpretation. Stating the denominator convention makes the density calculation unambiguous and reproducible.

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2.30 Density Interpretation

Density is a structural measure of connectivity. It does not measure elapsed time, note duration, tempo or the strength of an individual edge. It summarizes how many possible ordered key relationships are represented in the graph.

In the loop-inclusive representation, 20 of the 49 possible ordered pairs are active and 29 are inactive. In the loop-excluded representation, 17 of 42 non-loop ordered pairs are active and 25 are inactive. These counts are consistent because the three diagonal self-loop pairs are removed from both the numerator and denominator when loops are excluded.

The density values are close because the number of active self-loops is small relative to the total number of possible ordered pairs. The difference between 40.82% and 40.48% is the result of using different loop conventions, not a change in the musical data.

20 active	29 inactive

49 ordered pairs when loops are included

17 active	25 inactive

42 non-loop ordered pairs

Figure 9: Active and inactive directed pairs under the two density conventions

Definition 2.23. Density describes the proportion of possible directed relationships that occur in the network.

Example 2.26. With loops excluded, density is $17/42 \approx 0.40476$.

Key Point. Density is independent of how frequently an already-active edge occurs.

Interpretation. The density results indicate a moderately connected directed network with a substantial number of absent ordered pairs.

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2.31 Subgraph Analysis

A subgraph is formed by selecting a subset of vertices and retaining the edges that connect vertices within that subset. Subgraph analysis makes it possible to examine local transition structure without repeatedly displaying the complete seven-key network.

One useful two-node subgraph is formed by B and E. The matrix contains both $B \to E$ and $E \to B$, and also contains self-loops at each of these vertices. The local subgraph therefore includes reciprocal movement and persistence:

$$B \to E, \qquad E \to B, \qquad B \to B, \qquad E \to E.$$

The corresponding counts are 6, 2, 2 and 5, respectively. This shows that reciprocal transition does not require the two directions to occur equally often.

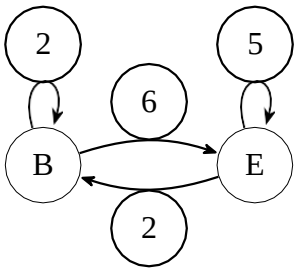


Figure 10: The two-key subgraph on B and E

Definition 2.24. A subgraph induced by a vertex set $U \subseteq V$ contains the selected vertices and every edge of the original graph whose endpoints both lie in U.

Example 2.27. The induced subgraph on $U = \{B, E\}$ contains the two reciprocal edges and the two self-loops shown above.

Key Point. Subgraphs preserve the original edge directions and counts among the selected vertices.

Interpretation. The B, E subgraph highlights local recurrence and the coexistence of reciprocal movement with self-transition.

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2.32 Three-Node and Four-Node Subgraphs

The transition matrix contains several useful local structures. The three-key set $\{A, G, F\#\}$ contains reciprocal links among its three vertices and supports two directed three-node cycles. The sequence $A \to G \to F\# \to A$ is one orientation, and $A \to F\# \to G \to A$ is the reverse orientation.

The three-key set $\{D, E, F\#\}$ contains the directed cycle

$$D \to E \to F\# \to D.$$

The set $\{D, C\#, B\}$ contains the directed chain

$$D \to C\# \to B,$$

but it does not form a cycle because $B \to D$ is absent.

A useful four-key subgraph is {E, F#, G, A}. It contains reciprocal transitions between E and F#, F# and G, G and A, and A and F#. The induced subgraph is strongly connected, although not every pair of vertices is directly adjacent.

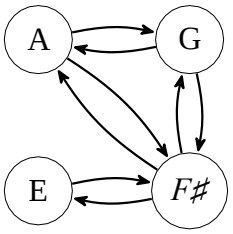


Figure 11: Interconnected four-key subgraph on $E, F\#, G, A$

Key Point. The matrix supports two directed cycles on the set $\{A, G, F\#\}$, one cycle on $\{D, E, F\#\}$, and a directed chain on $\{D, C\#, B\}$.

Interpretation. Local subgraphs reveal recurring patterns that are less visible in the full adjacency matrix.

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2.33 Subgraph Classification

The principal local structures can be classified according to their number of vertices and transition pattern. The classification below is based on positive entries of the seven-by-seven count matrix.

Subgraph	Vertices	Main structure	Type
SG1	B, E	$B \leftrightarrow E$ with self-loops at both vertices	Reciprocal two-node
SG2	$A, G, F\sharp$	Two directed three-node cycles	Cyclic three-node
SG3	$E, F\sharp, G, A$	Several reciprocal transitions in a connected core	Interconnected four-node
SG4	$D, E, F\sharp$	$D \to E \to F\sharp \to D$	Directed three-node cycle
SG5	$D, C\sharp, B$	$D \to C\sharp \to B$	Directed three-node chain

Table 1: Classification of selected musical subgraphs

The table distinguishes a reciprocal pair from a longer directed cycle and from a chain. A chain is not classified as a cycle because it does not return to its starting vertex. Likewise, a connected four-node subgraph may contain multiple cycles and reciprocal pairs without being a single simple cycle.

The classification is qualitative, while the transition matrix supplies the quantitative counts. Together, they describe both the local form and the frequency of movement within each selected substructure. The vertex sets are not mutually exclusive: a key such as $F\sharp$ belongs to several of the selected subgraphs because it participates in transitions with multiple keys.

Definition 2.25. Subgraph classification groups selected induced subgraphs according to their internal directed structure.

Example 2.28. SG4 is a three-node cycle because the final edge $F\sharp \to D$ returns to the starting vertex D.

Key Point. Reciprocal, cyclic, chain and interconnected structures are different graph types.

Interpretation. The classification table summarizes recurring local structures without changing the definitions of the original network.

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2.34 Overall Network Measures

The complete seven-key network can be summarized through vertex count, transition frequency, active edge count, density, self-loops, reciprocal pairs, cycles, centrality, PageRank and spectral radius. The measures below are derived from the transition-count matrix and the binary support graph under the conventions specified in the relevant sections.

Measure	Result	Interpretation
Distinct vertices	7	Seven musical keys
Note occurrences	113	Expanded note events
Transition occurrences	106	Consecutive directed movements
Active directed pairs	20	Includes three self-loop pairs
Active non-loop pairs	17	Directed edges excluding loops
Possible ordered pairs	49	Seven-by-seven, loops allowed
Density, loops included	0.40816	40.82 percent
Density, loops excluded	0.40476	40.48 percent
Distinct self-loop pairs	3	At A, B and E
Self-loop occurrences	9	Sum of diagonal counts
Reciprocal two-node pairs	7	Both directions active
Directed three-node cycles	3	Direction-sensitive cycle count
Largest PageRank	0.21085	At F#
Largest directed betweenness	0.2778	At F#
Largest outward closeness	0.7500	At F#
Spectral radius of $W = C/106$	0.20810	Largest absolute eigenvalue

Table 2: Consolidated measures of the seven-key transition network

The table combines measures that operate on different representations. Counts and strengths use the weighted transition matrix. Density and shortest paths use the binary support graph. PageRank uses the row-stochastic transition matrix. The spectral radius is calculated from the globally normalized matrix. These distinctions ensure that each result is interpreted according to the mathematical object from which it was obtained.

Key Point. The network summary contains both frequency-based and topology-based measures.

Interpretation. No single value describes the complete musical structure; the measures are complementary rather than interchangeable.

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2.35 Unified Interpretation of the Musical Network

The seven-key transition network provides a formal description of the ordered musical material. The seven vertices preserve the note identities, while the directed edges preserve the direction of consecutive transitions. The weighted counts retain the frequency of repeated movements. The model therefore contains the sequence information needed for a reproducible graph analysis.

The count matrix records 106 consecutive transitions among 113 note occurrences. Twenty ordered key-pairs are active, including three self-loop pairs. The loop-inclusive density is 40.82 percent, while the loop-excluded density is 40.48 percent. Seven reciprocal two-node pairs and three directed three-node cycles are present in the matrix. These values describe the topology of the active transition structure.

The global count-normalized strengths show that *F♯* and *G* share the largest outgoing value, while *F♯* has the largest incoming value. The full-network directed betweenness and outward closeness calculations also place *F♯* first. Weighted PageRank assigns *F♯* the largest stationary importance value, followed by *G* and *E*. The spectral radius of the global matrix $W = C/106$ is approximately 0.20810.

The temporal analysis remains a separate but related layer. The BPM formulas convert beat values into elapsed time, and the supplied timeline locates each numbered line within the passage. This temporal description should not be confused with the count matrix: transition counts do not themselves express note durations.

Key Point. The model combines note identity, directed succession, transition frequency, path structure and rhythmic timing while keeping their definitions distinct.

Interpretation. The results show how one selected musical passage can be studied through several compatible mathematical representations.

2.36 Final Results of the Seven-Key Musical Transition Network

The seven-key graph is defined by the vertex set $V = \{A, B, C♯, D, E, F♯, G\}$ and the transition-count matrix *C* displayed in Section 2.7. The row and column totals both equal 106, confirming the total number of consecutive transitions. The expanded note sequences contain 113 note occurrences.

The main numerical results are as follows:

[1.] The graph contains 7 vertices, 20 active directed key-pairs and 106 transition occurrences.

[2.] Three distinct self-loop edges occur at *A*, *B* and *E*, with 9 self-loop occurrences in total.

[3.] There are 7 reciprocal two-node pairs and 3 direction-sensitive simple cycles of length three.

[4.] Directed density is 0.40816 when self-loops are included and 0.40476 when they are ex-cluded.

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[5.] The outgoing strength maximum is shared by *F♯* and *G*, each with 0.24528; the largest incoming strength is 0.25472 at *F♯*.

[6.] The largest full-network betweenness is 0.2778 at *F♯*, and the largest outward closeness is 0.7500 at *F♯*.

[7.] PageRank with damping factor 0.85 gives the largest value, 0.21085, to *F♯*.

[8.] The spectral radius of $W = C/106$ is approximately 0.20810.

The results show that *F♯* is the leading vertex under incoming strength, PageRank, directed betweenness and outward closeness. It shares the maximum outgoing strength with *G*. The five-key chain remains a useful explanatory example, but the complete seven-key values above are the principal graph results.

Key Point. The principal hub in the complete seven-key network is *F♯*, with a tie between *F♯* and *G* in outgoing strength.

Conclusion

This study has developed a mathematical representation of the seven supplied musical lines through note sequences, rhythmic timing, and directed graph theory. The seven -key vocabulary A, B, C♯, D, E, F♯, and G forms the vertex set, and every consecutive pair of notes within a numbered line defines a directed transition. Repeated material in Lines 5 and 6 is expanded according to the supplied repeat indications, including the transition at each repeat join. The resulting data contain 113 note occurrences and 106 consecutive transitions. The transition-count matrix and its row and column totals provide consistent checks of this count basis.

The temporal framework converts tempo into elapsed time using the beat-duration formula 60/BPM. Under the stated one-beat and equal-subdivision condition, a group of n equal notes has a per-note duration of 60/(BPM × n). The comparisons at 60 BPM and 120 BPM show that doubling the tempo halves the duration of a beat and of each corresponding equal subdivision. The supplied timeline locates the musical segments within the passage. These temporal values describe timing; they do not alter the order or frequency of the directed note transitions.

The complete seven-key transition network contains 20 active directed key-pairs, including three self-loop pairs, and 17 active non-loop pairs. The self-loops occur at A, B, and E, with nine occurrences in total. The matrix supports seven reciprocal two-node pairs and three direction-sensitive three-node cycles. Directed density is 0.40816 when self-loops are included and 0.40476 when they are excluded. These results describe the topology of the observed transition network.

The strength and centrality results identify F♯ as the leading vertex under several distinct criteria. F♯ has the largest incoming strength, 0.25472, while F♯ and G share the largest outgoing strength, 0.24528. In the complete seven-key

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graph, F♯ has the largest normalized directed betweenness, approximately 0.2778, and the largest outward closeness, 0.7500. Weighted PageRank with damping factor 0.85 also assigns the largest value to F♯, approximately 0.21085. The spectral radius of the globally normalized matrix W = C/106 is approximately 0.20810. The five-key chain remains an illustrative subnetwork and is not substituted for the full seven-key results.

The findings show why several graph measures are needed. Strength records direct transition activity; betweenness describes a vertex's role on shortest directed paths; outward closeness measures access to the other vertices; and PageRank captures recursive importance under the row-stochastic transition model. The binary support graph, globally normalized count matrix, and row-stochastic PageRank matrix serve different purposes, and their results must be interpreted according to their respective definitions.

An important methodological distinction is maintained between rhythmic time and transition frequency. Tempo-based duration is calculated from the beat value and the stated subdivision assumptions, whereas the graph-count model records the frequency and direction of ordered note movements. These quantities may be combined only through a separately defined time-weighting rule; they are not interchangeable. Keeping them distinct makes the analysis transparent and reproducible.

Within the scope of the seven supplied lines and the stated counting conventions, the study provides a reproducible procedure for converting ordered musical note data into a directed mathematical network and evaluating its structural properties. It quantifies transition activity, connectivity, recurrence, and centrality while retaining the identities of the notes and the direction of musical succession. The framework offers a basis for further mathematical music analysis, provided that future applications state their source transcription, rhythmic assumptions, graph definitions, and normalization procedures explicitly.

References

[1.] Brin, S., & Page, L. (1998). The anatomy of a large-scale hypertextual web search engine. A. *Computer Networks and ISDN Systems, 30*(1–7), 107–117. https://doi.org/10.1016/S0169-7552(98)00110-X

[2.] Mauch, M., MacCallum, R. M., Levy, M., & Leroi, A. M. (2015). The evolution of popular music: USA 1960–2010. *Royal Society Open Science, 2*(5), 150081. https://doi.org/10.1098/rsos.150081

[3.] Newman, M. E. J. (2010). *Networks: An introduction.* Oxford University Press.

[4.] Serra, J., Corral, Á., Boguñá, M., Haro, M., & Arcos, J. L. (2012). Measuring the evolution of contemporary Western popular music. *Scientific Reports, 2*, 521. https://doi.org/10.1038/srep00521